Journal of Scientific Research and Development 2 (10): 5-8, 2015
Available online at www.jsrad.org
ISSN 1115-7569

Portfolio optimization for index tracking problem with mixed integer programming model

Lam Weng Siew [*], Lam Weng Hoe

Centre for Business and Management, Department of Physical and Mathematical Science, Faculty of Science, Universiti Tunku Abdul Rahman, Perak Campus, Jalan Universiti, Bandar Barat, 31900 Kampar, Perak

Abstract: Index tracking is a portfolio management which aims to construct an optimal portfolio to achieve similar mean return with the benchmark index means return without purchasing all the benchmark index components. The index tracking problem can be represented by an optimization model to minimize the tracking error. The objective of this paper is to construct an optimal portfolio to track the benchmark index return using the mixed integer programming model. In this study, the data consists of stock prices from Malaysia main market index which is FTSE Bursa Malaysia Kuala Lumpur Composite Index. The results of this study indicate that the optimal portfolio of mixed integer programming model is able to track FBKLCI index effectively at minimum tracking error 0.3963% which is closer to zero. Therefore, the mixed integer programming model is appropriate for the investors in Malaysia. The significance of this paper is to find out that the optimal portfolio generates higher mean return than FBMKLCI index mean return due to positive deviation from the benchmark index return.

Key words: Index tracking; Optimal portfolio; Benchmark index; Mean return; Tracking error

1. Introduction

Index tracking is a portfolio management which aims to construct an optimal portfolio to achieve similar mean return with the benchmark index means return without purchasing all the stocks from the benchmark index (Roll, 1992). Tracking error is a risk measure of how closely the portfolio follows the benchmark index return. The index tracking problem is represented by an optimization model in constructing the optimal portfolio. Roll (1992) introduced the optimization model in index tracking problem to minimize the tracking error. Various optimization models in index tracking problem have been developed and further studied by other researchers in different stock markets (Beasley et al., 2003; Canakgoz & Beasley, 2008; Guastaroba & Speranza, 2012; Lam et al., 2013; Lam et al., 2014a; Lam et al., 2014b; Lam et al., 2014c; Lam et al., 2014d; Lam et al., 2015a; Saiful & Lam, 2014; Wu et al, 2007; Saiful et al., 2015; Lam et al., 2015b; Lam et al., 2015c; Lam & Lam 2015; "in press" (Lam et al., a); "in press" (Lam et al., b)). The rest of the paper is structured as follows. The next section discusses about the data and methodology. Section 3 discusses about the empirical results of this study. Section 4 concludes the paper.

2. Data and methodology

FTSE Bursa Malaysia Kuala Lumpur Composite Index (FBMKLCI) is the leading indicator of the performance of the Malaysia stock market and economy which consists of 30 stocks listed on the Malaysian Main Market. In this study, the data consists of weekly price of 23 stocks in FBMKLCI index from January 2010 until December 2013. These 23 stocks are selected in the study since they make up as components of FBMKLCI consistently within the study period. This data is applied in the mixed integer programming model (Guastaroba & Speranza, 2012) for portfolio construction. The model is solved with branch and bound algorithm and computed using LINGO software.

2.1. Mixed integer programming model

Guastaroba and Speranza (2012) introduced a mixed integer programming model for index tracking problem. This model is an optimization model which aims to minimize the deviation of the value between the optimal portfolio and the benchmark index in order to minimize the tracking error. The mixed integer programming model is formulated as below.

Minimize $$z = \sum_{t=1}^{T} \left| \theta . I_t - \sum_{j=1}^{n} q_{jt} . w_j \right| \quad (1)$$

Subjects to

$$\theta = \frac{C}{I_T} \quad (2)$$

[*] Corresponding Author.

$$\sum_{j=1}^{n} y_j = K \quad (3)$$

$$\sum_{j=1}^{n} q_{jT} \cdot w_j = C \quad (4)$$

$$x_j = \frac{q_{jT} \cdot w_j}{C} \quad (5)$$

$$\sum_{j=1}^{n} x_j = 1 \quad (6)$$

$$y_j \in \{0,1\} \quad (7)$$

$$L_j y_j \le x \le U_j y_j \quad (8)$$

$$0 < L_j < U_j < 1 \quad (9)$$

$$x_j, w_j \ge 0 \quad (10)$$

T is the number of periods, I_t is the value of benchmark index at time t, C is the total amount invested at time T, q_{jt} is the price of one unit of stock j at time t, w_j represents the number of units for stock j in tracking portfolio. K is number of stocks selected to track the benchmark index, x_j is the weight of each stock invested, L_j and U_j are the lower and upper bounds of the investment proportion respectively on stock j.

Equation 1 is the objective function which aims to minimize the deviation of the value between the optimal portfolio and the benchmark index in order to minimize the tracking error. Constraint 2 is the parameter used to scale the value of the benchmark index. Constraint 3 ensures that the number of stocks in the optimal portfolio is equal to K. In this study, K is set to 10 (Beasley et al., 2003; Canakgoz & Beasley, 2008; Guastaroba & Speranza, 2012). Constraint 4 to constraint 6 ensures that total weight of stocks invested is equal to one. Constraint 7 and constraint 8 show that if stock j is not selected in the optimal portfolio (i.e., $y_j = 0$), then $x_j = 0$, and if stock j is selected in the optimal portfolio (i.e., $y_j = 1$), then $x_j \neq 0$. Constraint 9 indicates that the value of x_j is limited in the interval $[L_j, U_j]$. Constraint 10 states that the weight of each stock invested and the number of units of stock j in the portfolio are non-negative.

2.2. Tracking error

Roll (1992) introduced tracking error in index tracking problem to reproduce the performance of the benchmark market index without purchasing all stocks in the index. Tracking error is a risk measure of how closely the optimal portfolio follows the benchmark index. Variance is applied as a risk measure for tracking error in index tracking. Variance is the most commonly used risk measure in portfolio optimization (Markowitz, 1952). Tracking error is the standard deviation of the difference between the returns of the portfolio and the returns of the benchmark index (Meade & Salkin, 1990). The formula for tracking error is as follows.

$$TE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \left(R_{Pt} - R_{It}\right)^2} \quad (11)$$

TE is the tracking error; T is the number of periods, R_{Pt} is the mean return of the portfolio at time t and R_{It} is the mean return of the benchmark index at time t.

2.3. Portfolio mean return

The mean return of the portfolio is formulated as follow.

$$R = \sum_{j=1}^{n} R_j x_j \quad (12)$$

x_j is the weight of each stock invested and R_j is the mean return of stock j.

Excess return is defined as the difference between portfolio mean return and benchmark index mean return. There is only excess return if the portfolio mean return is higher than benchmark index mean return. Excess return is formulated as follow.

$$\alpha = r_P - r_I \quad (13)$$

α is the excess return, r_P is the mean return of the portfolio, r_I is the mean return of the benchmark index.

3. Empirical results

Table 1 presents the optimal portfolio composition which is constructed using the mixed integer programming model (Guastaroba & Speranza, 2012).

Table 1: Optimal portfolio composition of mixed integer programming model

Stock	Weights (%)
AMMB Holdings	0.00
Axiata Group Bhd	6.92
British American Tabaco	0.00
CIMB Group Holding	13.61
Digi.Com	0.00
Genting Bhd	4.35
Genting Malaysia	0.00
Hong Leong Bank Bhd	0.00
Hong Leong Financial Group	0.00
IOI Corporation	0.00
Kuala Lumpur Kepong	7.82
Malayan Banking	10.89
Maxis Bhd	4.48
Petronas Dagangan Bhd	7.82
Petronas Gas Bhd	0.00
Public Bank Bhd	22.75
PPB Group Bhd	0.00
RHB Capital Bhd	0.00
Sime Darby	16.96
Telekom Malaysia Bhd	0.00
Tenaga Nasional	4.40
UMW Holdings	0.00
YTL Corporation	0.00

As shown in Table 1, the list of stocks with positive weights indicates that those stocks are only selected by the mixed integer programming model in constructing the optimal portfolio. The optimal portfolio consists of 10 stocks with different weights to track FBMKLCI index which comprises 30 stocks. This implies that 33% of FBMKLCI index components are required in constructing the optimal portfolio to track the benchmark index. Public Bank Bhd is the most dominant stock in the optimal portfolio with 22.75% of the investment. Genting Bhd has the smallest component in the optimal portfolio with 4.35% of the investment.

Table 2 displays the portfolio performance of the mixed integer programming model.

Table 2: Portfolio performance of the mixed integer programming model

Portfolio	FBMKLCI Index	Optimal Portfolio
Weekly Mean Return (%)	0.1922	0.2165
Tracking Error (%)	-	0.3963
Weekly Excess Return (%)	-	0.0243

As shown in Table 2, the weekly mean return for FBMKLCI index is 0.1922% (annual mean return 10.50%) based on the study period. The optimal portfolio is tracking FBMKLCI index with weekly mean return 0.2165% (annual mean return 11.90%) at minimum tracking error 0.3963%. The tracking error is closer to zero which implies that the optimal portfolio is able to track FBMKLCI index effectively. Besides that, the positive deviation from benchmark index indicates the optimal portfolio generates weekly excess return 0.0243% (annual excess return 1.40%) over FBMKLCI index.

4. Conclusions

This paper discusses about the construction of the optimal portfolio in tracking Malaysia stock market index using the mixed integer programming model. The minimum tracking error is 0.3963% which is closer to zero. This implies that the optimal portfolio is able to track the benchmark index effectively with positive deviation. Therefore, the mixed integer programming model is suitable for the investors in Malaysia for index tracking. The significance of this study is to find out that the optimal portfolio generates weekly excess return 0.0243% over FBMKLCI index at minimum tracking error 0.3963%.

References

G. Guastaroba, and M. G. Speranza (2012). Kernel Search: An application to index tracking problem. European Journal of Operational Research, vol. 217, 54-68, 2012.

H. J. Saiful and W. S. Lam (2014). Portfolio Construction in Enhanced Index Tracking Modelling. International Conference on Business Strategy and Social Sciences 2014 (ICBSSS 2014), Handbook of Business Strategy and Social Sciences, pp. 260-263, 2014.

H. J. Saiful, W. S. Lam, and I. Hamzun (2015). A New Optimization Model in Enhanced Index Tracking Problem with Goal Programming Approach and Entropy Measure. Wulfenia Journal, vol. 22(1), Jan 2015.

H. M. Markowitz (1952). Portfolio Selection. Journal of Finance, vol. 7, pp. 77-91, 1952.

J. E. Beasley, N. Meade, and T. J. Chang (2003). An evolutionary heuristics for the index tracking problem. European Journal of Operational Research, vol. 148, pp. 621-643, 2003.

L. C. Wu, S. C. Chou, C. C. Yang, and C. S. Ong (2007). Enhanced Index Investing Based on Goal Programming. The Journal of Portfolio Management, vol 33, pp. 49-56, 2007

N. A. Canakgoz, and J. E. Beasley (2008). Mixed integer programming approaches for index tracking and enhanced indexation. European Journal of Operational Research, vol 196, pp. 384-399, 2008.

N. Meade, and G. R. Salkin (1990). Developing and Maintaining an Equity Index Fund. Journal of Operation Research Society, vol. 41(7), pp. 599-607, 1990.

R. Roll (1992). A Mean Variance Analysis of Tracking Error. Journal of Portfolio Management, vol. 18, pp. 13-22, 1992.

W. S. Lam, and W. H. Lam (2015). Portfolio Selection for Index Tracking Problem in Malaysian Stock Market. International Journal of Administration and Governance, vol. 1(3): 15-17, 2015.

W. S. Lam, H. J. Saiful, and I. Hamizun (2013). Enhanced Index Tracking Modelling in Portfolio Optimization. International Conference on Mathematical Sciences and Statistics 2013 (ICSMSS2013), AIP Conf. Proc., vol 1557, pp. 169-472, 2013.

W. S. Lam, H. J. Saiful, and I. Hamizun (2014a). Comparison between Two Stage Regression Model and Variance Model in Portfolio Optimization. Journal of Applied Science and Agriculture, vol. 9(18), pp. 36-40, 2014.

W. S. Lam, H. J. Saiful, and I. Hamizun (2014b). Index Tracking Modelling in Portfolio Optimization with Mixed Integer Linear Programming. Journal of Applied Science and Agriculture, vol. 9(18), pp. 47-50, 2014.

W. S. Lam, H. J. Saiful, and I. Hamizun (2014c). Enhanced Index Tracking Modelling in Portfolio Optimization with Mixed Integer Programming Approach. The 2014 UKM FST Postgraduate Colloquium, AIP Conf. Proc., vol. 1614, pp. 918-922, 2014.

W. S. Lam, H. J. Saiful, and I. Hamizun (2014d). Portfolio Optimization in Enhanced Index Tracking with Goal Programming Approach. The 2014 UKM FST Postgraduate Colloquium, AIP Conf. Proc., vol. 1614, pp. 968-972, 2014.

W. S. Lam, H. J. Saiful, and I. Hamizun (2015a). The impact of human behavior towards portfolio selection in Malaysia. Procedia of Social and Behavioral Sciences, vol. 172, pp. 674-678, 2015.

W. S. Lam, H. J. Saiful, and I. Hamizun (2015b), Investigation on relationship between human behavior and portfolio selection problem in Malaysia, Advances in Environmental Biology, vol. 9(7), pp. 6-10, 2015.

W. S. Lam, H. J. Saiful, and I. Hamizun (2015c), An empirical study on the characteristics on high risk aversion behavior in portfolio decision making, Advances in Environmental Biology, vol. 9(7), pp. 17-20, 2015.

W. S. Lam, H. J. Saiful, and I. Hamizun (a), “An empirical comparison of different optimization models in enhanced index tracking problem”, Advanced Science Letters, in press.

W. S. Lam, H. J. Saiful, and I. Hamizun (b), “The impact of different economic scenarios towards portfolio selection in enhanced index tracking problem”, Advanced Science Letters, in press.