Journal of Scientific Research and Development 1 (2): 34-37, 2014
Available online at www.jsrad.org
ISSN 1115-7569

Range based processing in modeling of dynamic behavior

Aisulu Akbota
Southern Federal University, Rostov-on-Don, Russia

Abstract: The paper proposes dynamical relations with microstructure, that generalize the known equations of gradient elasticity, used for analyzing static stress-stain condition of materials with dislocations and disclinations. It is shown that the considered relations extend the equations of motion in terms of displacements of the classical elasticity theory, passing in the limiting case of vanishing gradient terms to the dynamic Lame's equations. The equivalence of introduced equations to relations of Leroux continuum under certain ratios of the model parameters is proved. It is highlighted that based on proposed relations it is possible to describe the dynamics of multiscale processes (micro and nanostructures) through the segregation both regular components of wave fields corresponding to the results of the linear elasticity theory and boundary-layer components, that take into account the microstructural features of media.

Key words: Gradient elasticity ・ Continuum ・ Dynamic equations ・ Multiscale processes ・ Microstructural features

INTRODUCTION

To evaluate the stress-strain state of materials with nanodislocations and disclinations [1, 2] a special gradient model, proposed in [3] is widely used:

$$\sigma = \lambda(tr\varepsilon)I + 2\mu\varepsilon - c\nabla^2\left[\lambda(tr\varepsilon)I + 2\mu\varepsilon\right] \quad (1)$$

where [λ] and [μ] - Lamé coefficients, $\sigma = \{\sigma_{ij}\}$ and $\varepsilon = \{\varepsilon_{ij}\}$- tensors of elastic stresses and strains, I-ordinary tensor, ∇^2 - Laplasian operator, trε- trace of the strain tensor; $c>0$ –a gradient coefficient that, when being equal to zero, leads to degeneration of the equation (1) into Hooke's law of classical theory of elasticity.

Correct description of nonstationary processes in elastic media with nanostructured defects requires new dynamic models. These generalized relations are obtained excluding the strains in the equations of motion of the linear theory of elasticity [4].

$$\partial\sigma_{ji}/\partial x_j = \rho\,\partial^2 u_i/\partial t^2$$

Here the components of displacement vector $u = \{u_{ij}\}$ are associated with the components of the strain tensor in the well known equation $\varepsilon_{ij} = 0{,}5\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)$; [$\rho$] - material density, t - time and $i,j \in \{1,2,3\}$ and the summation is over the free index j.

Thus, basing on the relations of the gradient model (1) we derive the generalized dynamic equations of the displacement gradient elasticity (similar to Lame equations [4]), which describe the different multiscale nonstationary processes [5]:

$$\left(1 - c\nabla^2\right)\left[(\lambda+\mu)\nabla e + \mu\nabla^2\mathbf{u}\right] = \rho\frac{\partial^2\mathbf{u}}{\partial t^2} \quad (2)$$

The necessity of taking into account the displacement gradient of higher-order was first noted by Le Roux in [6]. The result (2), generalizing the models [1-3], is obtained based on the assumption of small deformations described by the linear gradient model, by analogy with the famous Le Roux continuum [6, 7]:

$$(\lambda+\mu)\,grad\,div\,\mathrm{u} + \mu\Delta\mathrm{u} - 4\mu M^2 signM\Delta\left(\Delta\mathrm{u} + \tilde{v}\,grad\,div\,\mathrm{u}\right) = \rho\frac{\partial^2\mathrm{u}}{\partial t^2} \quad (3)$$

The direct comparison of the dynamic relations (2) and (3) allows to assume that the proposed nonstationary gradient model (2) coincides with the model of Le Roux continuum (3) only in the case of certain correlation between Lame coefficients ([λ] and [μ]), the gradient coefficient c and the continuum constants ($M, \tilde{v} > 0$), namely $c = 4M^2 sign\,M,\quad \tilde{v} = 4(1 + \lambda/\mu)$.

Corresponding Author: Aisulu Akbota

The generalized equations of the gradient one-parameter model for dynamic behavior of materials with nanostructured defects allow particular simplifications.

Two-Dimensional Problem: In the case of two-dimensional nonstationary problem taking into account the standard notation given in (2)

$$\mathbf{u} \equiv \begin{Bmatrix} u_1 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} U \\ W \end{Bmatrix}, \; \mathbf{x} \equiv \begin{Bmatrix} x_1 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x \\ z \end{Bmatrix},$$

$$\nabla = \left\{ \frac{\partial \cdot}{\partial x}, \frac{\partial \cdot}{\partial z} \right\}^T, \; \nabla^2 \equiv \Delta \cdot = \frac{\partial^2 \cdot}{\partial x^2} + \frac{\partial^2 \cdot}{\partial z^2},$$

$$\mathrm{e} = \sum_{i=1}^{2} \varepsilon_{ii} = \frac{\partial U}{\partial x} + \frac{\partial W}{\partial z},$$

We obtain the scalar form of generalized Lame equations (2) in the two-dimensional case:

$$\begin{cases} (\lambda+\mu)\dfrac{\partial e}{\partial x} + \mu \nabla^2 U - c\Lambda_1 = \rho \ddot{U} \\ (\lambda+\mu)\dfrac{\partial e}{\partial z} + \mu \nabla^2 W - c\Lambda_3 = \rho \ddot{W} \end{cases}$$

$$\Lambda_1 = \mu \nabla^4 U + (\lambda+\mu)\left(\frac{\partial^4 U}{\partial x^4} + \frac{\partial^4 U}{\partial x^2 \partial z^2} \right) + (\lambda+\mu)\left(\frac{\partial^4 W}{\partial x^3 \partial z} + \frac{\partial^4 W}{\partial x \partial z^3} \right)$$

$$\Lambda_3 = \mu \nabla^4 W + (\lambda+\mu)\left(\frac{\partial^4 W}{\partial x^2 \partial z^2} + \frac{\partial^4 W}{\partial z^4} \right) + (\lambda+\mu)\left(\frac{\partial^4 W}{\partial x^3 \partial z} + \frac{\partial^4 W}{\partial x \partial z^3} \right) \tag{4}$$

Reaching the limit values in (2) and (4) at $c \to 0$, we derive the famous Lame equations in the framework of classical linear elasticity theory [4].

The nonsheared case (μ=0) gives

$$\begin{cases} \lambda\left(\dfrac{\partial e}{\partial x} - c\left\{ \dfrac{\partial^4 U}{\partial x^4} + \dfrac{\partial^4 U}{\partial x^2 \partial z^2} + \dfrac{\partial^4 W}{\partial x^3 \partial z} + \dfrac{\partial^4 W}{\partial x \partial z^3} \right\} \right) = \rho \ddot{U} \\ \lambda\left(\dfrac{\partial e}{\partial z} - c\left\{ \dfrac{\partial^4 W}{\partial z^4} + \dfrac{\partial^4 W}{\partial x^2 \partial z^2} + \dfrac{\partial^4 U}{\partial x^3 \partial z} + \dfrac{\partial^4 U}{\partial x \partial z^3} \right\} \right) = \rho \ddot{W} \end{cases} \tag{5}$$

At $c \to 0$ the dynamic relations (4), (5) degenerate into the corresponding well-known particular equations of elasticity theory.

The Antiplane Problem: In the case of antiplane nonstationary problems of uniaxial displacement of the medium (for example, only in the direction Oy: $u_2 = u(x,z,t)$, with the shear velocity defined as $\vartheta \equiv \vartheta_S = \sqrt{\mu/\rho}$, the vector equation of the generalized model (2) degenerates into a scalar ratio

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u - c\left(\frac{\partial^4 \cdot}{\partial x^4} + \frac{\partial^4 \cdot}{\partial z^4} + 2\frac{\partial^4 \cdot}{\partial x^2 \partial z^2} \right) u = \frac{1}{\vartheta^2} \frac{\partial^2}{\partial t^2} u \tag{6}$$

which results in wave equation at c=0 [4, 7, 8].

Here, with the type of progressive waves

$$u(x,z,t) = U(\alpha,\kappa)\exp(\gamma z + i\alpha x - i\omega t),$$

with the notation $\kappa = \omega/\vartheta = \omega\sqrt{\rho/\mu}$, we obtain a biquadratic equation $c\gamma^4 - (2c\alpha^2+1)\gamma^2 + (c\alpha^4 + \alpha^2 - \kappa^2) = 0$,
which has four roots, for defining the eigenvalue of γ:

$$\gamma_j = \pm\sqrt{\alpha^2 - \frac{-1 \pm \sqrt{1+4c\kappa^2}}{2c}} = \begin{cases} \pm\sqrt{\alpha^2 - \dfrac{-1+\sqrt{1+4c\kappa^2}}{2c}} & j=1,2 \\ \pm\sqrt{\alpha^2 - \dfrac{-1-\sqrt{1+4c\kappa^2}}{2c}} & j=3,4 \end{cases}. \tag{7}$$

At $c \to 0$ two eigenvalues

$$\gamma_j\big|_{c\to 0} = \pm 1/\sqrt{c} \cdot \left(1 + c(\alpha^2+\kappa^2)/2 + O(c^2\kappa^4)\right), \quad (j=3,4)$$

tend to infinity $|\gamma_j| \to \infty$, while the others reach the limit values which correspond to the antiplane shear problem:

$$\gamma_j\big|_{c\to 0} = \pm\sqrt{\alpha^2-\kappa^2} \cdot \left(1+O(c)\right), \quad (j=1,2).$$

Thus, general solution (5) for the displacement field of one-parameter gradient elasticity can be expresses as

$$u(x,z,t) = \exp(i\alpha x - i\omega t)\sum_{j=1}^{4} U_j(\alpha,\kappa) \cdot \exp(\gamma_j z) \tag{8}$$

Numerical and analytical methods for studying spatial-temporal structure of dispersive wave fields of type (8) (for problems with different initial-boundary conditions) have been developed and presented in [8 - 12].

From the asymptotic decompositions $\gamma_j\big|_{c\to 0}$ $(j=\overline{1,4})$, it follows that in the general solution (8) the first two terms (j = 1, 2) are regular constituents and at $c\to 0$ allow limit values of gradient elasticity which coincide with the general solution to the problem of antiplane shear of the classical theory of elasticity. The remaining terms (j = 3, 4), due to the gradient model, under natural conditions of solution limitations (8), that take the form

$$\left|\gamma_j z\right| = |z|/\sqrt{c}\cdot\left\{1+O\left(c(\alpha^2+\kappa^2)\right)\right\}_{c\to 0} \le const,\quad \alpha^2+\kappa^2 \le const,\quad (j=3,4)$$

determine the boundary layer component of displacement, which make a finite contribution to the solution of (8) at $c\to 0$ only for small values of z, thus describing the local (nano-and microstructure) scale of dynamic processes identified in the static problems of nanostructures [1, 2].

Thus, the obtained generalized dynamic equations of the continuum with a nanostructure (2) and their particular cases, derived from the equations of motion of linear elasticity model and a special gradient materials (1), allow us to describe different multiscale processes in materials with regard to their nanostructural features in nonstationary cases.

The Dynamic Antiplane Problem for a Semiplane: In the problem of antiplane shear for the medium ($-\infty < z \le 0,\quad -\infty < x < +\infty$) with the boundary conditions

$$\left\{\sigma^*_{32},\quad \partial\sigma^*_{32}/\partial z\right\}\Big|_{z=0} = \left\{f_1(x),\quad f_2(x)\right\}\cdot e^{i(\alpha x-\omega t)}$$

and the conditions of limit in (6)

$$z\to -\infty,\quad u(x,z,t)\to 0,$$

we obtain

$$u(x,z,t) = \mu^{-1}\left\{K_1(\alpha,z)f_1(x)+K_2(\alpha,z)f_2(x)\right\}\cdot e^{i(\alpha x-\omega t)}$$
$$\mathrm{Re}(\gamma_j)\ge 0,\quad (j=1,3). \qquad (9)$$

Where the functions $K_j\,(a,z)$ (j = 1,2) can be defined as

$$K_1(\alpha,\kappa,z) = -\frac{\gamma_3\cdot\Delta_3(\alpha,\kappa)\cdot e^{\gamma_1 z}-\gamma_1\cdot\Delta_1(\alpha,\kappa)\cdot e^{\gamma_3 z}}{\Delta(\alpha,\kappa)},$$
$$K_2(\alpha,\kappa,z) = \frac{\Delta_3(\alpha,\kappa)\cdot e^{\gamma_1 z}-\Delta_1(\alpha,\kappa)\cdot e^{\gamma_3 z}}{\Delta(\alpha,\kappa)},$$
$$\Delta_j(\alpha,\kappa) = \gamma_j\left[1+c\left(\alpha^2-\gamma_j^2\right)\right];\quad (j=1,3)$$
$$\Delta(\alpha,\kappa) = (\gamma_1-\gamma_3)\cdot\Delta_1(\alpha,\kappa)\cdot\Delta_3(\alpha,\kappa),\quad \kappa=\omega/\vartheta=\omega\sqrt{\rho/\mu}. \qquad (10)$$

In (9) and (10), the roots of $K_j^{-1}(\alpha,z)=0;\quad (j=1,3)$ determine the dispersion curves of wave numbers [α] of the frequency [κ], which is reduced to the necessity of solving the functional equation $\Delta(\alpha,\kappa)=0$.

The dispersion curves $\alpha=\alpha(\kappa,c)$ at for various values of the gradient coefficient c (c1=10^{-2}, c2=10^{-3}, c3=10^{-4}, c4=10^{-5}) are shown in Figure 1.

It is easy to prove, that the character of decreasing the amplitudes of wave fields (9) with increasing frequency on top boundary (z=0) of the medium can be expresses as

$$K_1(\alpha)=K_1(\alpha,0)\sim\kappa^{-5/2},\quad K_2(\alpha)=K_2(\alpha,0)\sim\kappa^{-2},\quad \kappa\to\infty \qquad (11)$$

Figure 2 illustrates the amplitude-frequency dependence (e.g. modules) for different values of the gradient coefficient c (c1=10^{-1}, c2=10^{-3}, c3:= 2 10^{-4}, c4:=10^{-6}) and α=100.

Resonances of the amplitude (10) due to dispersion are seen to superimpose on the power-law downward at infinity trend (11). This amplitude singularity ($K_1^{-1}(\alpha,\kappa_*)=0$) is taken place at frequencies corresponding to the inverse values of the dispersion relation $\kappa_*\equiv\kappa(\alpha)=\alpha\cdot\sqrt{1+c\cdot\alpha^2}$.

The power-law character of the wave amplitude decay that is recognized on the basis of asymptotic expansion (11) is visually obvious.

Accounting gradient terms are seen to lead to a decrease in wave numbers (in addition to the appearance of boundary-layer contributions to the solution, considering the microstructure influence) and an increase in the wavelength (Fig. 1). A growing of the gradient factor value leads to a shift in the amplitude resonances

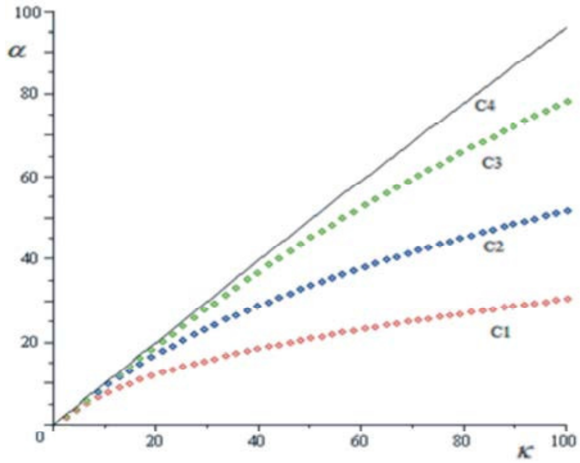

Fig. 1: The dispersion curves for various values of the gradient coefficient *c*.

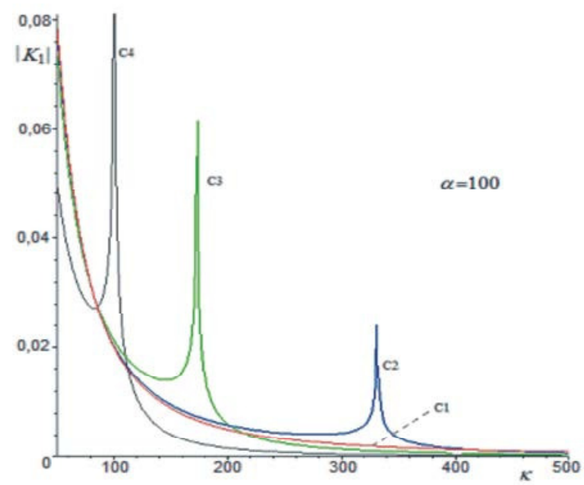

Fig. 2: The amplitude-frequency dependence for the gradient coefficient *c*.

to higher frequencies (Fig. 2) and a lowering of wave attenuation effect at large frequencies κ>400 due to $|K_1|_{c=c1} > |K_1|_{c=c2} > \cdots$.

REFERENCES

1. Gutkin, M. Yu. and I.A. Ovid'ko, Plastic Deformation in Nanocrystalline Materials / M.Yu. Gutkin, I.A. Ovid'ko, 2004. Berlin-Heidelberg-N.Y.: Springer, pp: 197.
2. Gutkin, M. Yu. and E.C. Aifantis, 1999. Dislocations and disclinations in a gradient theory of elasticity. Physics of the Solid State, 41(12): 2158-2166.
3. Altan, S.B. and E.C. Aifantis, 1992. On the structure of the mode III crack-tip in gradient elasticity. Scripta Metal. Mater., 26(2): 319-324.
4. Novatsky, V., 1985. Theory of asymmetrc elasticity. N.Y.: Pergamon Press, pp: 384.
5. Zolotarev, A.A., 2010. Materials dynamics simulation with regard to their nanostructural features. RusNanotech. Matireals of Nanotechnology International Forum. Moscow 2010, (November 1-3, Expocenter), CD: Rusnanotech/content/ en/2green/ file111.pdf, ISBN 978-5-9902492-1-9.
6. Le Roux. 1911. Etude geometrique de la torsion et de la flexion. Ann. Scient. De L'Ecole Normale Sup. Paris, 28: 523-579.
7. Erofeyev, V.I., 2003. Wave processes in solids with microstructure. New Jersey – London: World Scientific Publishing. pp: 256.
8. Babeshko, V.A., A. A. Zolotarev, A.A. Ivanov and G.V. Tkachev, 1984. Investigation of wave fields in a fluid layer and elastic half-space excited by a near-bottom source of vibrations./ J. Applied Mechanics and Technical Physics, 25(4): 542-544.
9. Zaitseva, I.A. and A.A. Zolotarev, 1993. On the space–time structure of wave field in a double–layer medium with an interface crack. J. Appl. Maths Mechs, 57(1): 197-204.
10. Zolotarev, A.A., 1990. One approach to solving integral equations of the initial-boundary value problems for layered media. News of the USSR Academy of Sciences. Solid State Mechanics, 6: 30-35.
11. Andrejchikova, I.I. and A.A. Zolotarev, 1993. Waves diffraction by a thin inclusion within a two-layer waveguide. Acoustical Physics, 39(5): 773-781.
12. Zolotarev, A.A., 2001. Asymtotic analysis of wave fields when there is partial impulse delamination of the media. J. Appl. Maths Mechs, 65(1): 139-144.