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Profound Understanding of fundamental mathematics (PUFM) among K-5/6 mathematics teachers

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Abstract: Helping all students develop a high level of mathematical proficiency needs teachers who can teach mathematics in great depth. "Depth" does not only mean making all students master arithmetic procedures, but also making them realize why a certain procedure works. For this matter, the teachers should possess deeper understanding of conceptual knowledge and procedural knowledge. This study assessed the K-5 and K-6 mathematics teachers' profound understanding of mathematics. Further, the study investigated if a successful completion of a college education would guarantee a thorough understanding of mathematics. Twenty-nine teachers were included in the study. Qualitative and quantitative research methods were used for this study. Survey was conducted in order to collect the data while mean and regression analysis were used to analyze the data. The results indicate that non- mathematics majors are as good as mathematics majors both in the conceptual and procedural knowledge.

Key words: Profound Understanding, Conceptual Knowledge, Procedural Knowledge

1. Introduction

Results of the 2003 Third International Mathematics and Science Study (TIMMS) show that the Philippines ranked 34th out of the 38 countries participating in the survey. In 2008, even with only science high schools participating the Philippines ranked lowest among ten (10) countries in the advanced mathematics category. These findings are quite alarming considering the fact that many "new math" movements have been introduced to improve the mathematics performance of our pupils. Nationwide, it is a common observation among college mathematics teachers that majority of the freshmen are not fully equipped with basic concepts, computational and problem-solving skills. It is prevalent among college mathematics teachers to put the blame of this trend to the high school and elementary mathematics teachers; while the high school mathematics teachers, in turn, would also point their fingers to the elementary mathematics teachers. In most cases, college mathematics teachers teach topics which are supposed to be learned in the elementary and high school levels; and the high school mathematics teachers teach topics which are supposed to be learned in the elementary level.

Several factors could be attributed to this very poor performance of our pupils. One of these is the mathematics teachers' mathematical knowledge and teaching strategies. It has been observed that a lot of mathematics teachers have focused on procedural knowledge in teaching, concentrating on algorithms and formulae with less or no emphasis on conceptual knowledge. Generally, most teachers and laymen, view elementary mathematics as simple mathematics which actually demands a deep understanding of the basic ideas or topics such as addition, subtraction, multiplication, division and place value.

Since it is in the elementary grades that good foundation in mathematics is established, it is essential that elementary mathematics teachers should be competent both in conceptual knowledge and procedural knowledge. It is on this premise that the researchers decided to conduct this study, that is, to assess the teachers' profound understanding of fundamental mathematics. It investigated if a successful completion of a college education would guarantee a thorough understanding of mathematics.

2. Literature review

This study was anchored on a constructivist perspective on learning which defines constructivism as a belief that all knowledge is a product of one's cognitive act by building on previous knowledge that allows one to move to new knowledge (Lerman, 1996). Clearly, a pupil's previous knowledge influences the process of incorporating new ideas and concept into already developed and existing structures. Constructivists, like Piaget, Vygotsky and Dubinsky, agreed that knowledge is not acquired automatically, but the

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learner should construct his or her own understanding. If pupils are aware of what they are doing and can express themselves in relation with their experiences, then understanding occurs and maybe lasting and easy to recall. They need the guidance of knowledgeable teachers in order to avoid formation of misconceptions and inadequacy of conceptual understanding. But this can only be realized if mathematics teachers exhibit a deep grasp of concepts in mathematics, not just knowing more mathematics and the procedures. In other words, their learning goal is "know how, and also know why" (Ma, 1999 as cited by Roger Howe).

In the study of Ma (1999) in which this study was derived, she made a comparison of the profound understanding of fundamental mathematics of elementary mathematics teachers in China and the United States. She addressed four (4) domains of fundamental mathematics or arithmetic: a) subtraction with renaming, b) multidigit multiplication, c) division by fractions and d) relationship between perimeter and area. The present study was also based on the domains used by Ma. She describes a teacher with Profound Understanding of Fundamental Mathematics (PUFM) as not only aware of conceptual structure & basic attitudes of mathematics in elementary mathematics, but also able to teach them to students. Her significant findings shows two main differences among Chinese and American mathematics teachers. These two (2) principal differences she mentioned were as follows: (a) Chinese teachers are specialists, and teach only mathematics, but American teachers, being generalists, are required to teach a range of different subjects. (b) In China, mathematics teachers are allocated a good amount of time for preparation of lessons, and they operate within a national curriculum that offers clear guidelines and relevant teaching materials. Conversely, U.S. teachers spend the whole day in front of a class. Working in relative isolation, they tend to rely too heavily upon commercially produced textbooks. She concluded that teachers will teach better if they are working from a meaningful knowledge base. The interactions between "what it is" and "how to teach it" seem to provide the driving force for the growth of the Chinese teachers' knowledge of school mathematics while collegiality collects momentum for the process. Teaching materials are highly relevant to the process. Ma recommends changes in a) teacher support; b) teacher education; c) mathematics education research, if American (US) teachers are to develop PUFM.

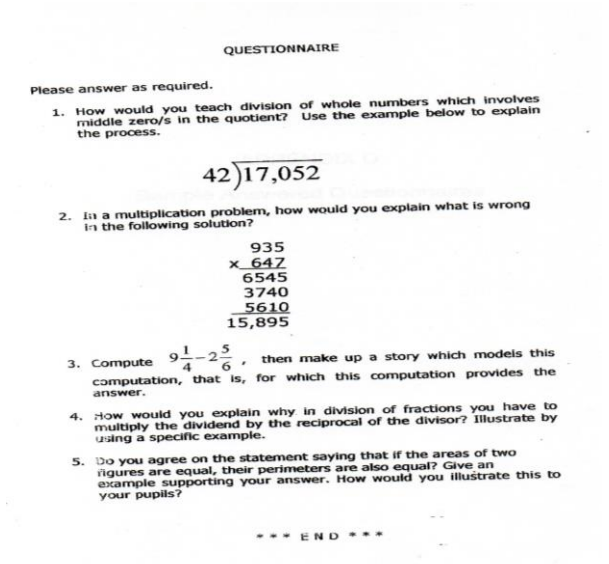
The present study determines the level of the PUFM in the context of K-5/6 mathematics teachers in the selected elementary schools in the division of Cagayan de Oro City and the possible implications in relation to pupil's performance in the national assessment comparison. Also, this study shows the baseline data for the extension services in the form of an enhancement program which the USTP can provide to these K-5/6 mathematics teachers.

3. Methodology

Both qualitative and quantitative research approaches were utilized in this research study. The data were collected by using a questionnaire and conducting informal interview. To determine the location of the participating schools, purposive sampling procedure was used to ensure that all school districts in the division of Cagayan de Oro City, Philippines are represented. When already identified, stratified random sampling was employed in the selection of the sample from each district.

Twenty-nine Grades 5 and 6 mathematics teachers from the public elementary schools in Cagayan de Oro City, Philippines were the participants of this study in which 69% of them come from urban schools. The age of the participants ranged from 29 to 64. Only 31% of them have mathematics as their field of specialization based on their college degree.

The teachers' grasp of mathematics was probed considering their answers to the five questions on the questionnaire and the ones asked during the informal interview. These five questions were organized around the topics for Grades 5 and 6 mathematics as recommended on the curriculum for elementary mathematics. The five open-ended questionnaire was shown in the Fig. 1.



QUESTIONNAIRE

Please answer as required.

1. How would you teach division of whole numbers which involves middle zero/s in the quotient? Use the example below to explain the process.

$42\overline{)17{,}052}$

2. In a multiplication problem, how would you explain what is wrong in the following solution?

935
× 647
6545
3740
5610
15,895

3. Compute $9\frac{1}{4}-2\frac{5}{6}$, then make up a story which models this computation, that is, for which this computation provides the answer.
4. How would you explain why in division of fractions you have to multiply the dividend by the reciprocal of the divisor? Illustrate by using a specific example.
5. Do you agree on the statement saying that if the areas of two figures are equal, their perimeters are also equal? Give an example supporting your answer. How would you illustrate this to your pupils?

*** END ***

Fig. 1: Open-Ended Questionnaire

Percentage, mean, standard deviation and regression analysis were used in the analysis of data.

4. Results and findings

Of the 29 teachers included in the study, 28% are beginners while 72% are experienced teachers. Only 31% of the Grades 5 and 6 mathematics teachers have mathematics as their field of specialization while 69% have either other subject areas as their field of specialization or had no specialization at all.

The contrast between the performances of the teachers in the procedural and conceptual

knowledge is very dramatic. Results show that teachers are strong in procedural knowledge but weak in conceptual knowledge. It is alarming to note that nobody gave a correct explanation in the division of fractions.

Table 1: Overall Performance

Topics	Conceptual		Procedural	
	f	%	*f*	%
Division of whole numbers	12	41	16	65
Multiplication of whole numbers	13	45	15	52
Subtraction of fractions	1	3	18	62
Division of fractions	0	0	14	48
Measurement	14	48	9	31

Table 2: Mean Score

Specialization	N	Conceptual		N	Procedural	
		$\bar{X}$	SD		$\bar{X}$	SD
Major	9	1.44	.88	8	3.50	1.31
Non-major	20	1.55	.94	20	2.85	1.35
Total	29	1.52	.91	28	3.04	1.35

There is a very poor mastery on conceptual knowledge of the Grades 5 and 6 mathematics teachers. Results show that on conceptual knowledge, the majors had a very low mean of 1.44 which is lower than the non-major who got a mean of 1.55. On procedural knowledge, the majors had a mean score of 3.50 which shows that they have an average understanding on the computational procedures. The non-majors had a mean score of 2.85 which is quite low considering that college degree holders are supposed to master the basic computational procedures. This shows that the majors have a better understanding on the computational procedures. For the non-majors, they are weak in both conceptual knowledge and procedural knowledge.

On the conceptual knowledge, results of the regression analysis identify one out of the six predictors to have contributed significantly to the Profound Understanding of Fundamental Mathematics (PUFM). The best predictor is the age of the teachers. The regression coefficient of -0.089 suggests that the young teachers have a better understanding than the older ones. This could be attributed to the fact that the pre-service training of the young teachers' emphasized learning mathematics using the conceptual approach compared with the old teachers whose pre-service training dealt more on procedural knowledge.

Results reveal that when taken jointly, the six predictors do not form a significant set of predictors for the Profound Understanding of Fundamental Mathematics as indicated by the F-value of 1.538.

On the procedural knowledge, results of the regression analysis show no significant predictors to the procedural knowledge. All of the predictors show weak or marginal influence. Taken jointly, the six predictors do not form a significant set of predictors as shown on the F-value of 0.740.

Table 3: Regression Analysis on Conceptual Knowledge

Predictors	*Regression Coefficient*	*Standard Error*	*t*	*Probability Value*
Field of Specialization	0.535	0.431	1.242	.230
School Type	-0.620	0.482	-1.287	.214
Rank	-1.076	0.576	-1.867	.078
Age	-0.089	0.24	-2.428	.026*
Sex	-0.085	0.591	-0.143	.888
Teaching Experience	0.145	0.535	0.271	.790
Constant	5.439	1.866	2.914	.009
Multiple R = .582; R^2=.339; Standard Error of the Estimate = 0.8571; F – value = 1.538; Probability = .222				

*p<.05

Table 4: Regression Analysis on Procedural Knowledge

Predictors	*Regression Coefficient*	*Standard Error*	*t*	*Probability Value*
Field of Specialization	-0.475	0.765	-0.621	.543
School Type	-0.873	0.771	-1.132	.273
Rank	-1.372	0.920	-1.491	.154
Age	-0.014	0.041	-0.353	.728
Sex	-1.028	0.949	-1.083	.294
Teaching Experience	1.303	0.861	1.513	.149
Constant	7.265	3.020	2.405	.028
Multiple R = .455; R^2=.207; Standard Error of the Estimate = 1.366; F – value = 0.740; Probability = .625				

*p<.05

4. Conclusions and Recommendations

The following conclusions were drawn:

a) Grades 5 and 6 mathematics teachers are fairly strong on procedural knowledge but are very weak on conceptual knowledge considering the fact that only 20% of the questions on conceptual knowledge were correctly answered.

b) Younger teachers are more competent in terms of knowing how a procedure works and why it works.
c) Pre-service training of elementary teachers is not sufficient to ensure thorough understanding of mathematics.
d) Some teachers are poor in communication skills which might be one of the causes of the misconceptions of the pupils.

The following recommendations are hereby forwarded:

a) Teachers teaching mathematics should undergo trainings/workshops in mathematics with more emphasis on conceptual knowledge acquisition.
b) Colleges/Universities offering education degrees should emphasize both conceptual knowledge and procedural knowledge in their mathematics courses. Elementary mathematics teachers should have mathematics as their field of specialization.
c) Mathematics teachers should attend relevant seminars which will enhance their communication skills.
d) A Professional Development Academy for elementary mathematics teachers should be established in every region.
e) College/Universities should conduct an extension activities on enhancing the profound understanding of fundamental mathematics of elementary mathematics teachers.
f) Education departments of countries lagging behind in the international comparisons of mathematics performance like TIMMS should also look into the status of their elementary mathematics teachers' profound understanding of fundamental mathematics and provide effective measures on how to improve their mathematics performance.

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