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A Comparison between the performances of synthetic and EWMA charts for monitoring the coefficient of variation

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Abstract: The coefficient of variation (CV) is a vital and widely used dimensionless measure of variability. The CV has many applications in scientific areas and is receiving growing attention in modern Statistical Process Control. It is well known that both the Synthetic and exponentially weighted moving average (EWMA) charts are very effective in detecting small to moderate changes in the CV. Therefore, a thorough comparison between the performances of these two optimal control charts is discussed in this paper. All the cases for detecting increasing and decreasing shifts in the nominal CV under both the zero- and steady-state modes are included in our study. The results reveal that the optimal EWMA CV-squared (EWMA-γ^2) chart outperforms the Synthetic CV (Syn-γ) chart for detecting small changes in the CV under the zero-state mode. However, for large increases in the CV, the Synthetic-γ chart is preferred. For the steady-state case, the EWMA-γ^2 chart is superior to the Synthetic-γ chart for detecting all levels of increases and decreases in the CV.

Key words: Average run length; Coefficient of variation; Control chart; Standard deviation of the run length

1. Introduction

Customer satisfaction is crucial in the world of businesses. Since quality products and services gain customers' satisfaction, improving quality and productivity are major factors leading to a successful business. Control charts are widely used in industrial and service sectors for detecting deterioration in product quality. Traditionally, a control chart cannot claim a normally distributed process as in-control unless the process has a constant mean and variance. However, there are many in-control processes where the mean is fluctuating and the variance is a linear function of the mean. In such conditions, the use of the traditional $\bar{X}$-type and S-type charts are dubious. Therefore, when the process mean and standard deviation are not constant at all the time, control charts for monitoring the sample coefficient of variation (CV) are used to keep track of the changes in the process mean or standard deviation. By definition, CV is the ratio of the standard deviation (σ) to the mean (μ), i.e. $\gamma = \sigma/\mu$. Since CV is unitless, it is generally used to compare data sets having different mean values or units.

The CV has many applications in the scientific areas. The CV has a salient application in the fields of manufacturing and materials engineering. For example, some properties of sintered materials and tool cutting life require the CV to be kept constant (Castagliola et al., 2011). In the field of finance, the CV is used as a tool to compare the risk-adjusted performance in mutual funds (Sharpe, 1966). Furthermore, it is used as the measurements for market risk (Marrison, 2002) and portfolio performance (Knight and Satchell, 2005). The CV is also adopted in health sciences to measure the geographic variation in the management of medication among the elderly (Zhang et al., 2010).

Kang et al. (2007) first introduced the Shewhart chart for monitoring the CV (denoted as Shewhart-γ). They applied the Shewhart-γ chart to monitor the cyclosporine level in organ-transplantation procedures. Castagliola et al. (2011) proposed the two one-sided exponentially weighted moving average (EWMA) charts of the CV squared (denoted as EWMA-γ^2). They applied this EWMA-γ^2 chart to monitor the pressure drop time related to the pore shrinkage in a metal sintering process. It is noted that Castagliola et al. (2011) only discussed the performance of the EWMA-γ^2 chart for the cases of increases and decreases in the CV under the zero-state mode. In 2013, Calzada and Scariano suggested a synthetic chart to monitor the CV (denoted as Syn-γ). They only studied the performance of the Syn-γ chart for the case of increases in the CV under the zero-state mode. Both EWMA-γ^2 and Syn-γ charts outperform the Shewhart-γ chart. Because of the advantages and wide usages of the CV, some new control charts for monitoring the CV have been proposed by many researchers recently. For example, Castagliola et al. (2013a, b) investigated the use of the variable-sampling-interval and selected run-rules features, respectively, to monitor the CV. Zhang et al. (2014) developed a new EWMA chart for

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monitoring the CV. The new EWMA procedure will not only use the information from the current sample, but also those from the former samples.

The existing literature ignores the steady-state case for both the Syn-γ and EWMA-γ^2 charts. In addition, the discussion for the decreases in the CV of the Syn-γ chart is excluded in the existing literature. Zero-state or steady-state run lengths is defined as the run lengths of control schemes initialized at the target value or evaluated after the control statistic has reached the steady state, respectively (Lucas and Saccucci, 1990). Therefore, in this paper, we make a thorough comparison of the performances between the Syn-γ and EWMA-γ^2 charts. The organization of this paper is as follows: Sections 2 and 3 review the Syn-γ and EWMA-γ^2 charts, respectively. Section 4 compares the performances in terms of the average run length (ARL) and standard deviation of the run length (SDRL) between these two optimal charts. Conclusions are drawn in Section 5.

2. The Synthetic-γ Chart

The Syn-γ chart comprises a Shewhart-γ chart and a standard confirming run length (CRL) chart (Calzada and Scariano, 2013). The plotting statistic for the Syn-γ chart is the sample CV ($\hat{\gamma}_i$), i.e.

$$\hat{\gamma}_i = \frac{S_i}{\bar{X}_i} \tag{1}$$

where S_i is the sample standard deviation, $\bar{X}_i$ is the sample mean and i = 1, 2, ... is the subgroup number. The upper (UCL) and lower (LCL) control limits for the Syn-γ chart are:

$$\text{UCL} = F_{\hat{\gamma}}^{-1}\left(1-\frac{p}{2}\middle| n, \gamma_0\right) \tag{2}$$

and

$$\text{LCL} = F_{\hat{\gamma}}^{-1}\left(\frac{p}{2}\middle| n, \gamma_0\right) \tag{3}$$

respectively, where n is the sample size, γ_0 is the in-control CV, $p = 1 - \Pr(\text{LCL} < \hat{\gamma}_i < \text{UCL})$ and $F_{\hat{\gamma}}^{-1}(\alpha|n,\gamma)$ is calculated as:

$$F_{\hat{\gamma}}^{-1}(\alpha|n,\gamma) ; \frac{\sqrt{n}}{F_t^{-1}\left(1-\alpha\middle| n-1, \sqrt{n}/\gamma\right)} \tag{4}$$

Here, $F_t^{-1}(\cdot)$ is the inverse cumulative distribution function (cdf) of the non-central t-distribution with non-centrality parameter $\sqrt{n}/\gamma$ and (n – 1) degrees of freedom.

The zero-state ARL and SDRL for the Syn-γ chart are computed as:

$$\text{ARL} = \mathbf{q}^T(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{1} \tag{5}$$

and

$$\text{SDRL} = \sqrt{2\mathbf{q}^T(\mathbf{I}-\mathbf{Q})^{-2}\mathbf{Q}\mathbf{1} - \text{ARL}^2 + \text{ARL}} \tag{6}$$

where $\mathbf{q} = (0,1,0,\ldots,0)^T$, $\mathbf{I}$ is the identity matrix and $\mathbf{1} = (1,1,\ldots,1)^T$. Here, the matrix $\mathbf{Q}$ of the transient probabilities is obtained through the Markov chain approach (Davis and Woodall, 2002). For example, when L = 3, where L is the lower limit of the CRL sub-chart in the Syn-γ chart, the matrix $\mathbf{Q}$ is equal to:

$$\mathbf{Q} = \begin{pmatrix} A & B & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \\ A & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

In Eq. (7), $A = F_{\hat{\gamma}}(\text{UCL}|n,\gamma_1) - F_{\hat{\gamma}}(\text{LCL}|n,\gamma_1)$ and $B = 1 - A$, where $F_{\hat{\gamma}}(x|n,\gamma)$ is approximated as:

$$F_{\hat{\gamma}}(x|n,\gamma) ; 1 - F_t\left(\frac{\sqrt{n}}{x}\middle| n-1, \frac{\sqrt{n}}{\gamma}\right) \tag{8}$$

and $\gamma_1 = \tau\gamma_0$ is the out-of-control CV. The values of $\tau \in (0, 1)$ correspond to a decrease in the nominal CV; whereas the values of $\tau > 1$ correspond to an increase in the nominal CV.

The steady-state ARL and SDRL are computed using the same formulae as in Eqs. (5) and (6), respectively, in which the initial probability vector $\mathbf{q}$ is replaced by the cyclical steady-state probability vector $\mathbf{q}_s$. This $\mathbf{q}_s$ can be calculated through the similar approach as discussed in Lucas and Saccucci (1990).

3. The EWMA-γ^2 Chart

The two one-sided EWMA-γ^2 charts are developed by Castagliola et al. (2011). The in-control mean $(\mu_0(\hat{\gamma}^2))$ and standard-deviation $(\sigma_0(\hat{\gamma}^2))$ of γ^2 are equal to:

$$\mu_0(\hat{\gamma}^2) = \gamma_0^2\left(1 - \frac{3\gamma_0^2}{n}\right) \tag{9}$$

and

$$\sigma_0(\hat{\gamma}^2) = \left[\gamma_0^4\left\{\frac{2}{n-1} + \gamma_0^2\left(\frac{4}{n} + \frac{20}{n(n-1)} + \frac{75\gamma_0^2}{n^2}\right)\right\} - \left(\mu_0(\hat{\gamma}^2) - \gamma_0^2\right)^2\right]^{1/2} \tag{10}$$

respectively. For the upward EWMA-γ^2 chart, the plotting statistic is defined as:

$$Z_i^+ = \max\left[\mu_0(\hat{\gamma}^2), (1-\lambda^+)Z_{i-1}^+ + \lambda^+\hat{\gamma}_i^2\right] \tag{11}$$

where $Z_0^+ = \mu_0(\hat{\gamma}^2)$ and the asymptotic UCL is equal to:

$$\text{UCL} = \mu_0(\hat{\gamma}^2) + K^+\sqrt{\frac{\lambda^+}{2-\lambda^+}}\sigma_0(\hat{\gamma}^2) \tag{12}$$

For the downward EWMA-γ^2 chart, the plotting statistic is defined as:

$$Z_i^- = \min\left[\mu_0(\hat{\gamma}^2), (1-\lambda^-)Z_{i-1}^- + \lambda^-\hat{\gamma}_i^2\right] \tag{13}$$

where $Z_0^- = \mu_0(\hat{\gamma}^2)$ and the asymptotic LCL is equal to:

$$\text{LCL} = \mu_0(\hat{\gamma}^2) - K^-\sqrt{\frac{\lambda^-}{2-\lambda^-}}\sigma_0(\hat{\gamma}^2) \tag{14}$$

Here, λ^+ (λ^-) and K^+ (K^-) are the smoothing constant and chart coefficient of the upward (downward) EWMA-γ^2 chart.

The zero-state ARL and SDRL for the upward and downward EWMA-γ^2 charts are computed by using Eqs. (5) and (6), respectively. Here, $\mathbf{q} = (1, 0, \ldots, 0)$ and **Q** is the (p+1, p+1) matrix of transient probabilities, where p is the number of subintervals between $\mu_0(\hat{\gamma}^2)$ and UCL or $\mu_0(\hat{\gamma}^2)$ and LCL for the upward or downward EWMA-γ^2 chart, respectively. The generic element $Q_{i,j}$, for i = 0, 1, ..., p, of the matrix **Q** can be calculated as follows (Castagliola et al., 2011):

- For the upward EWMA-γ^2 chart, if j = 0,

$$Q_{i,0} = F_{\hat{\gamma}^2}\left(\frac{\mu_0(\hat{\gamma}^2) - (1-\lambda^+)H_i}{\lambda^+}\middle| n, \gamma_1\right) \quad (15)$$

- For the downward EWMA-γ^2 chart, if j = 0,

$$Q_{i,0} = 1 - F_{\hat{\gamma}^2}\left(\frac{\mu_0(\hat{\gamma}^2) - (1-\lambda^-)H_i}{\lambda^-}\middle| n, \gamma_1\right) \quad (16)$$

- For both the upward and downward EWMA-γ^2 charts, if j = 1, 2, ..., p,

$$Q_{i,j} = F_{\hat{\gamma}^2}\left(\frac{H_j + \delta - (1-\lambda)H_i}{\lambda}\middle| n, \gamma_1\right) - F_{\hat{\gamma}^2}\left(\frac{H_j - \delta - (1-\lambda)H_i}{\lambda}\middle| n, \gamma_1\right). \quad (17)$$

where H_i or H_j, for i or j = 1, 2, ..., p is the midpoint of the i^{th} or j^{th} subinterval and H_0 = 0 represents the restart-state feature of the EWMA-γ^2 chart. Here, $\delta = (\text{UCL} - \mu_0(\hat{\gamma}^2))/2p$ and $\delta = (\mu_0(\hat{\gamma}^2) - \text{LCL})/2p$ for the upward and downward EWMA-γ^2 charts, respectively. Note that λ in Eq. (17) is either λ^+ or λ^- and $F_{\hat{\gamma}^2}(x|n,\gamma)$ in Eqs. (15) to (17) is defined as

$$F_{\hat{\gamma}^2}(x|n,\gamma) ; \; 1 - F_F\left(\frac{n}{x}\middle| 1, n-1, \frac{n}{\gamma^2}\right), \quad (18)$$

where $F_F(\cdot)$ is the cdf of the non-central F distribution with $(1, n-1)$ degrees of freedom and noncentrality parameter n/γ^2.

The steady-state ARL and SDRL are computed using the same formulae as in Eqs. (5) and (6), respectively, in which the initial probability vector **q** is replaced by the cyclical steady-state probability vector $\mathbf{q}_s$. These $\mathbf{q}_s$ can be calculated through the similar approach as discussed in Lucas and Saccucci (1990). Note that the matrix **Q** in Eqs. (5) and (6) are obtained from this section.

4. A Comparative study of the synthetic-γ and EWMA-γ^2 Charts

In this section, we compare the ARLs and SDRLs performance of the optimal Syn-γ and EWMA-γ^2 charts under both the zero- and steady-state modes. Tables 1 and 2 list the optimal charts' parameters (L, LCL, UCL) and (λ, K) of the Syn-γ and EWMA-γ^2 charts, respectively, in the first row of each cell. Note that λ^+ or λ^- and K^+ or K^- discussed in Section 3 are represented by λ and K in Tables 1 and 2. The charts' corresponding zero-state and steady-state (ARL_1, SDRL_1) values are presented in the second row of each cell in Tables 1 and 2, respectively. Here, ARL_1 and SDRL_1 represent the out-of-control ARL and SDRL, respectively. We consider the in-control ARL (ARL_0) = 500, n = 5, $\gamma_0 \in$ {0.1, 0.2} and $\tau \in$ {0.50, 0.65, 0.80, 1.25, 1.50, 1.75, 2.00} for all the cases studied in this paper. The optimization procedure discussed in Calzada and Scariano (2013) together with the formulae shown in Section 2, are used to obtain the optimal Syn-γ chart's parameters (L, LCL, UCL) and their corresponding (ARL_1, SDRL_1) values for both the zero- and steady-state cases. Meanwhile, concerning the EWMA-γ^2 charts, refer to Castagliola et al. (2011) for the optimization procedure and Section 3 for the related formulae. For example, when ARL_0 = 500, n = 5, γ_0 = 0.1 and τ =1.25 are desired, the optimal parameters (λ, K) = (0.0520, 2.9452) for the EWMA-γ^2 chart under the steady-state mode, are obtained through the written ScicosLab program. These optimal chart's parameters give the most minimum steady-state ARL_1 (=14.69) value among all the possible parameter combinations of the EWMA-γ^2 chart. Note that both the optimal Syn-γ and optimal EWMA-γ^2 charts' parameters must attain the zero- and steady-state ARL_0 = 500.

From Tables 1 and 2, we observe that the EWMA-γ^2 chart significantly outperforms the Syn-γ chart for detecting decreases in the CV under both zero- and steady-state conditions. For example, when τ = 0.65 and γ_0 = 0.1, the zero-state ARL_1 is 9.34 for the EWMA-γ^2 chart as opposed to 88.92 for the Syn-γ chart (see Table 1). This indicates that the EWMA-γ^2 chart is 89.5% on average faster than the Syn-γ chart in detecting a decrease of 0.65 in the CV when γ_0 = 0.1. This situation for the Syn-γ chart becomes worse under the steady-state mode. The zero- and steady-state SDRL_1 values for the EWMA-γ^2 chart are significantly smaller than that of the Syn-γ chart for all levels of decreasing shifts in the CV. For instance, when τ = 0.80 and γ_0 = 0.2, the steady-state SDRL_1 for the Syn-γ chart is 380.62; while that for the EWMA-γ^2 chart is 10.68 (see Table 2). This suggests that the run-length dispersion for the Syn-γ chart is remarkably higher than that of the EWMA-γ^2 chart.

Table 1: Optimal charts' parameters (L, LCL, UCL) and (λ, K) of the Syn-γ and EWMA-γ^2 charts, respectively, together with their corresponding zero-state (ARL_1, SDRL_1) values when ARL_0 = 500 and n = 5

τ	Syn-γ Chart (L, LCL, UCL) (ARL_1, SDRL_1)	EWMA-γ^2 Chart (λ, K) (ARL_1, SDRL_1)
$\gamma_0 = 0.1$		
0.50	(4, 0.02812, 0.18196) (17.31, 21.50)	(0.1500, 2.1645) (6.15, 1.00)
0.65	(9, 0.02537, 0.18834) (88.92, 107.89)	(0.2024, 2.1060) (9.34, 3.71)
0.80	(27, 0.02215, 0.19647) (364.76, 436.51)	(0.0735, 2.2065) (22.43, 10.91)
1.25	(35, 0.02147, 0.19830) (28.02, 35.40)	(0.0762, 3.1369) (16.47, 11.00)

1.50	(13, 0.02423, 0.19113) (6.32, 6.99)	(0.1500, 3.5366) (6.18, 3.95)
1.75	(7, 0.02618, 0.18640) (3.07, 2.95)	(0.2920, 3.9955) (3.56, 2.37)
2.00	(5, 0.02733, 0.18375) (2.07, 1.70)	(0.4008, 4.2343) (2.50, 1.63)
$\gamma_0 = 0.2$		
0.50	(4, 0.05580, 0.37536) (17.98, 22.31)	(0.1500, 1.9835) (6.07, 1.02)
0.65	(9, 0.05033, 0.38968) (91.95, 111.37)	(0.1500, 1.9835) (9.54, 3.28)
0.80	(27, 0.04394, 0.40810) (372.33, 445.05)	(0.0611, 1.9697) (22.67, 10.88)
1.25	(36, 0.04245, 0.41272) (29.65, 37.58)	(0.0782, 3.3705) (17.25, 11.55)
1.50	(13, 0.04807, 0.39597) (6.77, 7.63)	(0.1719, 3.8401) (6.50, 4.35)
1.75	(8, 0.05108, 0.38763) (3.28, 3.17)	(0.2677, 4.1655) (3.78, 2.53)
2.00	(6, 0.05298, 0.38259) (2.21, 1.82)	(0.3620, 4.4102) (2.66, 1.75)

Table 2: Optimal charts' parameters (L, LCL, UCL) and (λ, K) of the Syn-γ and EWMA-γ^2 charts, respectively, together with their corresponding steady-state (ARL_1, $SDRL_1$) values when ARL_0 = 500 and n = 5

τ	Syn-γ Chart (L, LCL, UCL) (ARL_1, $SDRL_1$)	EWMA-γ^2 Chart (λ, K) (ARL_1, $SDRL_1$)
$\gamma_0 = 0.1$		
0.50	(2, 0.03100, 0.17573) (22.18, 20.82)	(0.1500, 2.1680) (5.00, 1.47)
0.65	(3, 0.02946, 0.17900) (99.59, 98.26)	(0.1500, 2.1680) (7.99, 3.44)
0.80	(9, 0.02578, 0.18735) (373.95, 373.40)	(0.0538, 2.1954) (18.83, 10.27)
1.25	(22, 0.02326, 0.19358) (42.09, 38.35)	(0.0520, 2.9452) (14.69, 10.17)
1.50	(9, 0.02578, 0.18735) (10.95, 8.70)	(0.1500, 3.5410) (5.68, 3.91)
1.75	(6, 0.02706, 0.18436) (5.60, 3.80)	(0.1500, 3.5410) (3.39, 2.12)
2.00	(5, 0.02767, 0.18298) (3.86, 2.23)	(0.3020, 4.0226) (2.40, 1.53)
$\gamma_0 = 0.2$		
0.50	(2, 0.06153, 0.36151) (22.95, 21.58)	(0.1500, 1.9866) (4.99, 1.46)
0.65	(3, 0.05847, 0.36877) (102.76, 101.44)	(0.1587, 1.9800) (8.07, 3.63)
0.80	(9, 0.05115, 0.38745) (381.13, 380.62)	(0.0501, 1.9481) (19.17, 10.68)
1.25	(22, 0.04614, 0.40154) (44.25, 40.51)	(0.0537, 3.2002) (15.26, 10.61)
1.50	(10, 0.05052, 0.38917) (11.67, 9.31)	(0.1500, 3.7545) (5.97, 4.18)
1.75	(6, 0.05369, 0.38073) (5.96, 4.15)	(0.1500, 3.7545) (3.56, 2.28)
2.00	(5, 0.05490, 0.37764) (4.10, 2.46)	(0.2664, 4.1642) (2.55, 1.63)

For the case of increases in the CV, the zero-state (ARL_1, $SDRL_1$) values for the EWMA-γ^2 chart is smaller than those of the Syn-γ chart when 1.25 $\leq \tau \leq$ 1.50. However, for large increases in the CV (i.e $\tau \geq$ 1.75), the Syn-γ chart is superior to the EWMA-γ^2 chart under the zero-state mode. From Table 2, it is clear that the EWMA-γ^2 chart surpasses the Syn-γ chart for all levels of increasing shifts in the CV under the steady-state condition. The differences of the steady-state (ARL_1, $SDRL_1$) values for these two control charts are quite large especially for small increases in the CV. For example, when τ = 1.25 and γ_0 = 0.2, the steady-state (ARL_1, $SDRL_1$) values for the EWMA-γ^2 chart are (15.26, 10.61) and these values tremendously increase to (44.25, 40.15) for the Syn-γ chart (see Table 2).

It is apparent from Tables 1 and 2 that the steady-state (ARL_1, $SDRL_1$) values for the EWMA-γ^2 chart are lower than its zero-state (ARL_1, $SDRL_1$) values. However, the converse is true for the Syn-γ chart. We observe that the performances of the Syn-γ chart under the steady-state mode are considerably worse than its corresponding performances under the zero-state mode. For instance, the zero-state ARL_1 and $SDRL_1$ values for the Syn-γ chart are 28.02 and 35.40, respectively, when τ = 1.25 and γ_0 = 0.1 (see Table 1). These ARL_1 and $SDRL_1$ values increase by 50.2% and 8.3%, respectively, to 42.09 and 38.35 under the steady-state mode (see Table 2).

5. Conclusions

The CV control charts have been extended to a variety of non-traditional applications, for example, in the areas of finance, health care, education and many societal applications. There are many situations where neither the process standard deviation nor mean is constant, but the standard deviation is proportional to the mean. In such setting, the chart for monitoring the CV is a potentially attractive SPC tool.

In this paper, we make a thorough comparison of the ARL and SDRL performances between the optimal Syn-γ and optimal EWMA-γ^2 charts. All the cases including increases and decreases in the CV under both the zero- and steady-state conditions are considered in our study. For the zero-state case, the results reveal that the optimal EWMA-γ^2 charts perform better than the optimal Syn-γ chart for detecting all ranges of decreasing shifts and small increasing shifts in the CV. For large increases in the CV, we still opt for the optimal Syn-γ chart. On the other hand, the optimal EWMA-γ^2 charts remarkably outperform the optimal Syn-γ chart under the steady-state condition for all levels of decreasing and increasing shifts in the CV. In conclusion, the practitioners can select their preferred optimal control chart to monitor the CV for their specified and desired situations.

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