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Reliability estimation in Pareto-I distribution based on progressively type II censored sample with binomial removals

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Abstract: In this study, we deal with the estimation problem for the parameters and reliability characteristics; reliability function, hazard rate function and mean time system to failure, of Pareto-I distribution based on progressively type-II censored sample with random removals. The number of units removed at each failure time is assumed to follow a binomial distribution. The maximum likelihood method is used to obtain the estimators of the parameters and reliability characteristics functions of Pareto-I distribution. Monte Carlo simulation is performed to compare the performance of maximum likelihood estimates under progressively type-II censoring with the different random schemes.

Key words: Pareto-I distribution; Maximum likelihood method; Progressive type-II censoring; Binomial removals; Reliability characteristics

1. Introduction

The Pareto distribution has been widely used in the analysis of lifetime data from reliability, survival, insurance, economy and engineering, and so on (Johnson et al., 1994). Also, it is well known that in lifetime testing experiments, the failure times of all units placed on the test are not always observed by the experimenter. Samples that result from such cases are called censored samples. There are several types of censoring schemes. However, progressive censoring schemes in the last few years have been studied rather extensively by many authors. Because these schemes allow the experimenter to remove units before the termination of the experiment. Therefore, progressive censoring schemes are commonly used in reliability experiments, clinical trials, life-testing experiments, etc. For a comprehensive recent review of progressive censoring, see Balakrishnan and Aggarwala (2000).

One of the common progressive censoring schemes is progressive type-II censoring was introduced by Cohen (1963). The progressively type-II censored life test is defined as follows. The experimenter places n identical units on test at time zero and completely observe only m failures. When the first failure is observed, R_1 of the remaining $n-1$ surviving units are randomly selected and removed. Then after the second observed failure, R_2 of the remaining $n-1-R_1$ surviving units are randomly selected and removed, and so on. Finally, the experiment terminates until the m^{th} failure is observed and remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ surviving units all removed. If $R_1 = R_2 = \dots = R_{m-1} = 0$, then $R_m = n - m$ that corresponds Type-II censoring. If $R_1 = R_2 = \dots = R_m = 0$, then $n = m$ that corresponds the complete sample. Moreover, $R_1, R_2, \dots, R_m$ are all prefixed in this censoring scheme. However, in some practical situations, these numbers cannot be prefixed and they occur at random. For example, Yuen and Tse (1996) pointed out that the number of patients that withdraw from a clinical test at each stage is random and cannot be prefixed. Therefore, the statistical inference on lifetime distributions under progressive type II censoring with random removals has been studied in recent years by various authors, including Yuen and Tse (1996), Wu et.al. (2007), Yan et al. (2011), Dey and Dey (2014) and Azimi et. al. (2014).

In this study, we consider the two-parameter Pareto distribution of the first kind (Pareto I) as lifetime distribution. Also, for the case that the observed data are from the Pareto I distribution based on the progressive type II censoring with binomial removals, we deal with the estimation problems of the parameters and reliability characteristics such as reliability function, hazard rate function and mean time to failure.

In the literature, there are some studies on the inference of the Pareto distribution under progressive type II censoring with random removals. For instance, Wu and Chang (2003) studied the estimation problem for Pareto I distribution with one parameter based on progressive censoring with uniform removals. They used the maximum likelihood (ML) method to obtain the estimator of parameter. Wu (2003) provided the inference for the estimation of the two-parameter Pareto I

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distribution under progressive censoring with uniform removals. The ML method was also used for the estimation procedure of parameters. Amin (2008) considered the estimation and prediction problems using the Bayesian approach for the Pareto I distribution based on type-II progressive censoring with binomial removals. Shanubhogue and Jain (2012) obtained the uniformly minimum variance unbiased estimator for powers of the shape parameter and its functions of Pareto I distribution with known scale parameter under progressive type II censored data with binomial removals.

However, these authors were studied the problem for parameter(s) estimation of Pareto I distribution. In this paper, we consider the estimation problem for not only two parameters but also reliability characteristics of Pareto I distribution under progressive type II censored sample with binomial removals. The ML method is used to obtain the estimators of the parameters and reliability characteristics functions of Pareto-I distribution. Moreover, Monte Carlo simulation is performed to compare the performance of ML estimates under progressively type-II censoring with the different random schemes.

The remainder of this paper is organized as follows. In Section 2, the properties of Pareto-I distribution, reliability function, hazard-rate function and mean to system failure, are briefly presented. The ML estimators are derived under type II progressive censoring with binomial removals in Sections 3 and 4. The results of the simulation study are presented in Section 5. Section 6 summarizes the conclusions of the study.

2. The model

The Pareto distribution was firstly proposed Pareto (1897) as a model for the distribution of income but is now used as a model in a wide range of fields such as insurance, business, economics, engineering, survival and reliability.

Let the lifetime of a unit, X, have a Pareto-I distribution with the shape and scale parameters. The probability density function (pdf) of Pareto-I distribution is given by

$$f(x;\alpha,\beta)=\alpha\beta^{\alpha}x^{-(\alpha+1)} \quad ,x>\beta>0,\alpha>0 \tag{1}$$

where α and β are the shape and scale parameters, respectively. The corresponding cumulative distribution function (cdf) is given by

$$F(x;\alpha,\beta)=1-\beta^{\alpha}x^{-\alpha},\ x>\beta>0,\alpha>0 \tag{2}$$

Some reliability characteristics of Pareto-I distribution, the reliability function ($R(x)$), the hazard rate function ($h(x)$) and the mean time to system failure (MTSF) are expressed, respectively, as

$$R(x)=1-F(x)=\beta^{\alpha}x^{-\alpha},\quad x>\beta>0,\alpha>0 \tag{3}$$

and

$$h(x)=\frac{f(x)}{R(x)}=\beta^{\alpha}x^{-1},\quad x>\beta>0,\alpha>0 \tag{4}$$

and

$$MTSF=E[X]=\alpha\beta/(\alpha-1),\quad \alpha>1 \tag{5}$$

where $E[X]$ is expected value of Pareto I distribution. More usefulness properties of the Pareto I distribution as a lifetime model were discussed in Kus and Kaya, (2007), Parsi et al., (2010 and Fua et al., (2012).

3. Estimation

Let $X_1<X_2<...<X_m$ be a progressively type II censored sample from Pareto-I distribution, where $m<n$ is pre-fixed before the test. For progressive type II censoring with a pre-determined number of removals $R=(R_1=r_1,...,R_{m-1}=r_{m-1})$, the conditional likelihood function can be written as(Cohen,1963):

$$L_1(\alpha,\beta;x|R=r)=A\prod_{i=1}^{m}f(x_i)(1-F(x_i) \tag{6}$$

where

$$A=n(n-r_1-1)...(n-\sum_{i=1}^{m-1}r_1+1) \tag{7}$$

Substituting Eqs. (1) and (2) into Eq.(6), the likelihood function is derived as

$$L_1(\alpha,\beta;x|R=r)=A(r)\prod_{i=1}^{m}\alpha\beta^{\alpha}x_i^{-(\alpha+1)}(\beta^{\alpha}x_i^{-\alpha})^{r_i} \tag{8}$$

Suppose that an individual unit being removed from life test at the i^{th} failure, $i=1,2,...,m-1$, is independent of the others but with same probability p. Then the number of units removed at each failure time follows a binomial distribution with parameters $n-m-\sum_{l=1}^{i-1}r_l$ and p. Thus,

$$P(R_1=r_1)=\binom{n-m}{r_1}p^{r_1}(1-p)^{n-m-r_1},\quad 0\le r_1\le n \tag{9}$$

and

$$P(R_i=r_i|R_{i-1}=r_{i-1},...,R_1=r_1)=\binom{n-m-\sum_{l=1}^{i-1}r_l}{}p^{r_i}(1-p)^{n-m-\sum_{l=1}^{i-1}r_l} \tag{10}$$

where $0\le r_i\le n-m-\sum_{l=1}^{i-1}r_l,\ i=1,2,...,m-1$.

Moreover, we presume that R_i is independent of X_i for all i. Accordingly, the likelihood function can be expressed as

$$L(\alpha,\beta,p;x,r)=L_1(\alpha,\beta;x|R=r)P(R=r) \tag{11}$$

where $P(R=r)$ is the joint probability distribution given by

$$\begin{aligned}P(R=r)=&P(R_1=r_1)P(R_2=r_2/R_1=r_1)\\&\times P(R_3=r_3/R_2=r_2,R_1=r_1)\times...\\&\times P(R_{m-1}=r_{m-1}/R_{m-2}=r_{m-2},...,R_1=r_1)\end{aligned} \tag{12}$$

and

$$P(R=r)=\frac{(n-m)!}{(n-m-\sum_{i=1}^{m-1}r_i)\prod_{i=1}^{m-1}r_i!}p^{\sum_{i=1}^{m-1}r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i} \tag{13}$$

Now, using Eqs. (8), (11) and (13), we can write the full likelihood function as

$$L(\alpha,\beta,p;x,r)=BL_1(\alpha,\beta)L_2(p) \tag{14}$$

where

$$L_1(\alpha,\beta)=\prod_{i=1}^{m}\alpha\beta^{\alpha}x_i^{-(\alpha+1)}(\beta^{\alpha}x_i^{-\alpha})^{r_i} \tag{15}$$

and

$$L_2(p)=p^{\sum_{i=1}^{m-1}r_i}(1-p)(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i} \tag{16}$$

and

$$B=\frac{A(n-m)!}{(n-m-\sum_{i=1}^{m-1}r_i)\prod_{i=1}^{m-1}r_i!} \tag{17}$$

It is clear that B does not depend on the parameters α,β and p; L_2 is independent of parameters α,β.

4. Maximum likelihood estimation

In this section, we obtain the maximum likelihood estimators (MLEs) of the parameters α, β, p and the reliability characteristics $R(x)$, $h(x)$, MTSF based on progressively type II censoring data with binomial removals.

As mentioned before, L_1,given in Eq. (15), does not include p. Therefore, the MLEs of α and β can be derived by maximizing Eq. (15) directly. Since this likelihood function is an increasing function of β, and therefore, the MLE of β is given by

$$\hat{\beta}_{mle}=X_1 \tag{18}$$

The MLE of α can be obtained by solving $\frac{d\log L_2(\alpha,\hat{\beta})}{d\alpha}=0$. Then, it is found as

$$\hat{\alpha}_{mle}=\frac{m}{\sum_{i=1}^{m}(r_i+1)\log(x_i)-n\log(\hat{\beta})} \tag{19}$$

Similarly, L_2 in Eq. (16) does not involve α and β. Therefore, the MLE of p can be derived by maximizing Eq. (16) directly. Solving $\frac{d\log L_2(p)}{dp}=0$ with respect to p, the MLE for p is given by

$$\hat{p}_{mle}=\frac{\sum_{i=1}^{m-1}r_i}{\sum_{i=1}^{m-1}r_i+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i} \tag{20}$$

Additionally, Once the MLEs of α and β are obtained as $\hat{\alpha}$ and $\hat{\beta}$. By using invariance property of the MLEs, the MLEs of $R(x)$, $h(x)$ and MTSF are derived, respectively, as

$$\hat{R}(x)=\hat{\beta}^{\hat{\alpha}}x^{-\hat{\alpha}},\quad x>0 \tag{21}$$

and

$$\hat{h}(x)=\hat{\beta}^{\hat{\alpha}}x^{-1},\quad x>0 \tag{22}$$

and

$$M\hat{T}SF=\hat{\alpha}\hat{\beta}/(\hat{\alpha}-1) \tag{23}$$

5. Simulation study

In this section, a Monte Carlo simulation study is conducted to compare the performance of the ML estimates derived in the previous sections for progressively type II censoring with the different random schemes.

5.1. Algorithm for generating progressively type II censored samples from Pareto I distribution

By using the algorithm given in Balakrishnan and Sandhu, (1995), firstly generate the number of progressive censoring, r_i, $i=1,2,\dots,m$ with binomial removals between step 1 and step 6. Then, the following steps are used generate progressively type II censored order statistics from Pareto-I distribution.

The steps are:

1. Specify the value of n.
2. Specify the value of m.
3. Specify the values of parameters α,β and p.
4. Generate a random number r_1 from $Binom(n-m,p)$.
5. Generate a random number r_i from $Binom(n-m-\sum_{l=1}^{i-1}r_l,p)$, for each i, $i=2,3,\dots,m-1$.
6. Set r_m according to the following relation $r_m=\begin{cases}n-m-\sum_{k=1}^{m-1}r_k & ,\ n-m-\sum_{k=1}^{m-1}r_k>0\\ 0, & o.w\end{cases}$
7. Generate m independent uniform $U(0,1)$, random variables, $W_1 W_2,\dots,W_m$.
8. For given values of the progressive scheme $R=(r_1,r_2,\dots,r_m)$, set $V_i=W_i^{(i+\sum_{l=m-i+1}^{m}r_l)^{-1}}$ for $i=1,2,\dots,m-1$.
9. Set $U_i=1-V_m V_{m-1}\cdots V_{m-i+1}$, $i=1,2,\dots,m$ then $U_1,U_2,\dots,U_m$ are ranked progressive censored sample of size m from $U(0,1)$ with binomial removals.
10. Finally, for given values of parameters α and β, we set $X_i=1-F^{-1}(U_i)=\left(\frac{1-U_i}{\beta^{\alpha}}\right)^{-1/\alpha}$. Then, $(X_1,X_2,\dots,X_m)$ is the progressive type II censored sample from the Pareto-I distribution with binomial removals.

5.2. Simulation design

The design of simulation study is outlined in the following.

i) Given the values of parameters α, β and p, the mission time x , sample size n, number of failures m, generate a progressively type II censored sample of size n with m failures using the algorithm given in Section 5.1. For each value of n=20, 30 and 50, the values of m are taken as $(m/n)\times100$ = 40%, 60%, and 80% and the value of p is considered as 0.3 and 0.7.

ii) Compute the ML estimates of parameters α, β, p and reliability characteristics $R(x)$, $h(x)$ and MTSF by using MLEs given in Section 4.

iii) Given α=3, β=2 and p=0.3, 0.7, x=2.035, and α=4, β=3 and p=0.3, 0.7, x=3.039, repeat first and second steps N times where N is taken as 10000.

iv) Compare the estimates of parameters and reliability characteristics with the true values of them by computing the bias and MSE defined as (Karishna and Kumar, 2011):

$$Bias = \gamma(\theta) - \frac{1}{N}\sum_{i=1}^{N}\hat{\gamma}(\theta_i) \quad (24)$$

and

$$MSE(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\gamma}(\theta_i) - \gamma(\theta))^2 \quad (25)$$

where $\hat{\gamma}(\theta_i)$, i=1,...,N, are N estimates of $\gamma(\theta)$ and N is the number of simulation replications.

It is note that all calculations are performed on the MATLAB.

5.3 Simulation results

The obtained results from the simulation study are reported as the bias and MSE of the ML estimators in Tables 1-2 for α=3, β=2 and α=4, β=3 under progressively type II censoring with binomial removals according to p=0.3 and 0.7.

The further results are summarized in Tables 3-4 for x=2.035, $R(x)=0.95$, $h(x)=1.474$, $MTSF=3$ and x=3.039, $R(x)=0.95$, $h(x)=1.316$, $MTSF=4$ based on progressively type II censoring with binomial removals according to p=0.3 and 0.7.

Table 1: Simulation results for α=3, β=2 and p=0.3, 0.7

p=0.3		$\hat{\alpha}$		$\hat{\beta}$		$\hat{p}$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	0.989	4.264	0.034	0.002	0.015	0.007
20	12	0.602	1.829	0.034	0.002	0.026	0.011
20	16	0.440	1.101	0.034	0.002	0.058	0.028
30	12	0.599	1.875	0.023	0.001	0.010	0.004
30	18	0.384	0.930	0.022	0.001	0.017	0.006
30	24	0.272	0.582	0.023	0.001	0.037	0.016
50	20	0.328	0.762	0.013	0.000	0.007	0.002
50	30	0.213	0.425	0.014	0.000	0.009	0.003
50	40	0.157	0.296	0.013	0.000	0.021	0.008
p=0.7		$\hat{\alpha}$		$\hat{\beta}$		$\hat{p}$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	1.008	4.267	0.034	0.002	0.018	0.012
20	12	0.602	1.824	0.034	0.002	0.026	0.018
20	16	0.422	1.096	0.034	0.002	0.048	0.034
30	12	0.590	1.773	0.022	0.001	0.012	0.008
30	18	0.375	0.904	0.022	0.001	0.018	0.012
30	24	0.291	0.590	0.022	0.001	0.032	0.024
50	20	0.337	0.778	0.013	0.000	0.006	0.005
50	30	0.210	0.423	0.013	0.000	0.010	0.007
50	40	0.157	0.300	0.014	0.000	0.021	0.015

Table 2: Simulation results for α=4, β=3 and p=0.3, 0.7

p=0.3		$\hat{\alpha}$		$\hat{\beta}$		$\hat{p}$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	1.352	7.563	0.038	0.003	0.016	0.007
20	12	0.823	3.403	0.037	0.003	0.024	0.011
20	16	0.570	1.922	0.038	0.003	0.059	0.029
30	12	0.790	3.163	0.025	0.001	0.011	0.004
30	18	0.500	1.603	0.025	0.001	0.017	0.007
30	24	0.377	1.061	0.025	0.001	0.037	0.016
50	20	0.473	1.454	0.015	0.000	0.007	0.002
50	30	0.280	0.753	0.015	0.000	0.011	0.004
50	40	0.217	0.527	0.015	0.000	0.020	0.008
p=0.7		$\hat{\alpha}$		$\hat{\beta}$		$\hat{p}$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	1.327	7.482	0.038	0.003	0.015	0.012
20	12	0.824	3.359	0.038	0.003	0.026	0.018
20	16	0.576	1.950	0.038	0.003	0.045	0.034
30	12	0.809	3.375	0.025	0.001	0.012	0.008
30	18	0.499	1.601	0.025	0.001	0.016	0.012
30	24	0.342	0.985	0.025	0.001	0.034	0.024
50	20	0.441	1.353	0.015	0.000	0.007	0.005
50	30	0.280	0.760	0.015	0.000	0.011	0.007
50	40	0.211	0.523	0.015	0.000	0.021	0.015

It can be seen from Tables 1 and 2 that for α and β, the biases and MSEs decrease as long as the sample size n and the failure information m increase under different choices of censoring random schemes according to p=0.3 and 0.7. Furthermore, for p=0.3, the bias and MSE values of parameters α and β are similar to the results obtained for p=0.7. On the other hand, for p, the biases and MSEs increase as n and m increase.

Table 3: Simulation results for x=2.035, $R(x)=0.95$, $h(x)=1.474$, $MTSF=3$ and p=0.3, 0.7

p=0.3		$\hat{R}(t)$		$\hat{h}(t)$		$M\hat{T}SF$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	0.050	0.009	0.486	1.030	0.032	0.538
20	12	0.051	0.007	0.296	0.442	0.009	0.227
20	16	0.050	0.007	0.216	0.266	0.002	0.157
30	12	0.030	0.003	0.294	0.453	0.019	0.240
30	18	0.031	0.003	0.189	0.225	0.007	0.147
30	24	0.031	0.002	0.134	0.141	0.002	0.101
50	20	0.016	0.001	0.161	0.184	0.015	0.125
50	30	0.018	0.001	0.105	0.103	0.005	0.078
50	40	0.018	0.001	0.077	0.071	0.002	0.059
p=0.7		$\hat{R}(t)$		$\hat{h}(t)$		$M\hat{T}SF$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	0.051	0.011	0.496	1.030	0.046	0.350
20	12	0.050	0.007	0.296	0.441	0.007	0.254
20	16	0.050	0.007	0.207	0.265	0.008	0.168
30	12	0.029	0.003	0.290	0.428	0.024	0.218
30	18	0.031	0.003	0.184	0.218	0.008	0.137
30	24	0.031	0.002	0.143	0.142	0.008	0.097
50	20	0.016	0.001	0.166	0.188	0.016	0.127
50	30	0.017	0.001	0.103	0.102	0.003	0.080
50	40	0.018	0.001	0.077	0.072	0.003	0.060

Table 4: Simulation results for x=3.039, $R(x)=0.95$, $h(x)=1.316$, $MTSF=4$ and p=0.3, 0.7

p=0.3		$\hat{R}(t)$		$\hat{h}(t)$		$M\hat{T}SF$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	0.051	0.009	0.445	0.819	0.067	0.223
20	12	0.049	0.007	0.271	0.369	0.026	0.160
20	16	0.050	0.007	0.187	0.208	0.005	0.119

30	12	0.029	0.003	0.260	0.343	0.039	0.154
30	18	0.031	0.003	0.165	0.174	0.016	0.103
30	24	0.031	0.002	0.124	0.115	0.007	0.077
50	20	0.016	0.001	0.156	0.158	0.030	0.092
50	30	0.017	0.001	0.092	0.082	0.008	0.060
50	40	0.018	0.001	0.072	0.057	0.005	0.044
$p=0.7$		$\hat{R}(t)$		$\hat{h}(t)$		$M\hat{T}SF$	
n	m	Bias	MSE	Bias	MSE	Bias	MSE
20	8	0.051	0.010	0.437	0.810	0.058	0.239
20	12	0.050	0.007	0.271	0.364	0.027	0.157
20	16	0.050	0.006	0.190	0.211	0.007	0.117
30	12	0.030	0.003	0.266	0.365	0.037	0.162
30	18	0.031	0.003	0.164	0.173	0.016	0.100
30	24	0.032	0.002	0.112	0.107	0.001	0.075
50	20	0.016	0.001	0.145	0.147	0.024	0.090
50	30	0.017	0.001	0.092	0.082	0.007	0.062
50	40	0.018	0.001	0.069	0.057	0.003	0.046

The results with regard to the bias and MSE in Tables 3 and 4 then point out that for the reliability characteristics; $R(x)$, $h(x)$ and $MTSF$, the biases and MSEs decrease once n and m increase under different choices of censoring random schemes according to p=0.3 and 0.7. Additionally, considering the biases and MSEs of the reliability characteristics for p=0.3, 0.7, the similar results are observed for both p values.

6. Conclusion

In this paper, we consider the estimation problems of not only two parameters but also reliability characteristics of two parameters Pareto I distribution under progressive type II censored sample with binomial removals. Moreover, Monte Carlo simulation is conducted to compare the performance of maximum estimates under progressively type-II censoring with the different random schemes.

As a consequence, the overall simulation results reveal that (i) the MLEs of shape parameter α are very good in terms of the bias and MSE for all censoring schemes. (ii) the MLEs are strongly suggested to estimate scale parameter β with regard to the bias and MSE for all censoring schemes. (iii) the point estimates with the ML method of the parameter p are so good in terms of the bias and MSE for all censoring schemes, but the estimates are worse as n and m increase. (iv) the MLEs of reliability characteristics, reliability function, hazard rate function and mean time system to failure, give the satisfactory results in terms of the bias and MSE for all censoring schemes.

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