Journal of Scientific Research and Development 2 (12): 101-107, 2015
Available online at www.jsrad.org
ISSN 1115-7569

Comparison of different estimation methods for the Marshall–Olkin extended Weibull distribution

Ilhan USTA [1, *], Eda Celik [2]

[1]Doctor of Philosophy, Associate Professor, Department of Statistics, Faculty of Science, Anadolu University, Eskisehir, Turkey
[2]Master of Statistics, Faculty of Science, Anadolu University, Eskisehir, Turkey

Abstract: The aim of this paper is to determine an efficient method for estimating the parameters of Marshall–Olkin extended Weibull distribution. For this purpose, the performance of different estimation methods called maximum likelihood, least-squares and weighted least-squares methods, is compared in terms of the bias and mean square error through an extensive numerical simulation. It can be concluded from simulation results that the least-square method usually shows good performance in terms of bias for the considered cases. Moreover, with regard to the mean square error, the weighted least square method unexpectedly outperforms the other estimators for most of the considered cases.

Key words: Marshall–Olkin extended Weibull; Maximum likelihood; Least-squares; Weighted least-squares; Simulation

1. Introduction

In recent years, the Weibull distribution has been one of the commonly used distributions for modeling data from various scientific fields such as reliability, engineering, survival, finance and so on. However, the Weibull distribution cannot represent all situations found in many applications. One of the reasons is that the hazard rate function of Weibull distribution is unable to exhibit a bathtub shaped. Therefore, several researchers have been investigated to increase the flexibility of Weibull distribution in the last decease or so. Adding parameters to a well-defined distribution has been accepted as a good technique to obtain more flexible distributions.

Marshall and Olkin (1997) proposed a general method of adding a parameter into a baseline distribution. The obtained distribution, called Marshall–Olkin (MO) extended distribution, includes the baseline distribution as a special case and provide more flexibility to model various types of data. Let $\bar{F}(x) = 1 - F(x)$ be a baseline survival function of a continuous random variable X that depends on an q-dimensional parameter vector $\theta = (\theta_1, \theta_2, \ldots, \theta_q)^T$. Then, the survival function of MO extended distribution is defined by

$$\bar{G}(x) = \frac{\gamma\bar{F}(x)}{1-(1-\gamma)\bar{F}(x)} = \frac{\gamma\bar{F}(x)}{F(x)+\gamma\bar{F}(x)} \qquad (1)$$

where $-\infty < x < \infty$ and $\gamma > 0$ is called tilt parameter.

Clearly, the baseline survival function $\bar{F}(x)$ is obtained by γ=1. The probability density function (pdf) and hazard rate function corresponding to Eq. 1 are derived, respectively, as

$$g(x) = \frac{\gamma f(x)}{[1-(1-\gamma)\bar{F}(x)]^2} \qquad (2)$$

and

$$h(x) = \frac{r(x)}{1-(1-\alpha)\bar{F}(x)} \qquad (3)$$

where f(x) and r(x) denote the baseline pdf and the baseline hazard rate function, respectively.

In literature, some special cases of MO extended distribution are obtained by using well-known distributions as the baseline distributions such as Weibull and exponential (Marshall and Olkin, 1997), Pareto (Ghitany, 2005), gamma (Risti´c et al., 2007), Fréchet (Krishna et al., 2013) and Lindley (Espirito Santo and Mazuchelib, 2014) distributions. Also, Souza et al. (2013) presented some general results for MO extended distribution.

In this paper, we deal with estimation of the three parameters of MO extended Weibull (MOEW) distribution, suggested by Marshall and Olkin (1997). In literature, the MOEW distribution has been studied by various authors. For example, Ghitany et al. (2005) presented that the MOEW distribution can be obtained as a compound distribution with mixing exponential distribution. Zhang and Xie (2007) examined the model characterization based on the Weibull probability plot. Gupta et al. (2010) studied the effect of the tilt parameter on the monotonicity of the failure rate

* Corresponding Author.

and estimated the turning points of the failure rate of the extended Weibull distribution. Srivastava and Kumar (2011) estimated the two-parameter of the MOEW distribution using two method, maximum likelihood estimate and Bayes estimate and compared them. Cordeiro and Lemonte (2013) studied some mathematical properties of MOEW distribution. Also, they determined the moments of the order statistics and discussed the estimation of the parameters using maximum likelihood method. Santo-Neto (2014) introduced a new class of models called the MOEW family of distributions based on the work by Marshall and Olkin.

However, these authors are considered only the maximum likelihood (ML) method to estimate the three parameters of MOEW distribution and do not compare the MLE method with other estimation methods commonly used literature. In this paper, we consider three different methods, the ML method, the least-squares (LS) method, the weighted least-squares (WLS) method, to estimate the three parameters of MOEW distribution. Furthermore, the performance of these estimation methods is compared in terms of bias, mean square error (MSE) through an extensive numerical simulation.

The remainder of this paper is organized as follows. In Section 2, the properties of MOEW such as density function, hazard-rate function and moments are presented. The considered estimators are given in Section 2. The results of the simulation study are presented in Section 3. Section 4 summarizes the conclusions of the study.

2. Marshall-Olkin extended Weibull distribution

The three parameter MOEW distribution is generated by adding a new parameter into two-parameter Weibull distribution as baseline distribution. In Eq. 1, considering the survival function of two-parameter Weibull distribution, given as follows

$$\bar{F}(x)=e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \tag{4}$$

Then, the survival function of three parameter MOEW distribution is expressed as

$$\bar{G}(x)=\frac{\gamma e^{-\left(\frac{x}{\alpha}\right)^{\beta}}}{1-(1-\gamma)e^{-\left(\frac{x}{\alpha}\right)^{\beta}}} \tag{5}$$

where $0<x<\infty$, $\alpha>0$ is the scale parameter, $\beta>0$ is the shape parameter and $\gamma>0$ (Zhang and Xie, 2007). Hence, the corresponding cumulative distribution function (cdf) is

$$G(x)=1-\bar{G}(x)=\frac{1-e^{-\left(\frac{x}{\alpha}\right)^{\beta}}}{1-(1-\gamma)e^{-\left(\frac{x}{\alpha}\right)^{\beta}}} \tag{6}$$

The probability density and hazard functions of MOEW distribution are given, respectively,

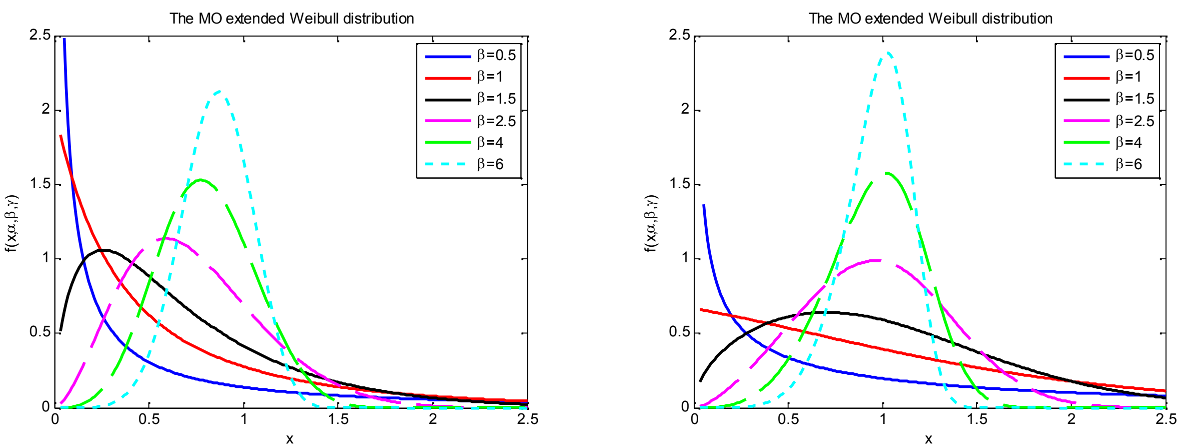
$$g(x)=\frac{\gamma\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}e^{-\left(\frac{x}{\alpha}\right)^{\beta}}}{\left[1-(1-\gamma)e^{-\left(\frac{x}{\alpha}\right)^{\beta}}\right]^{2}} \tag{7}$$

and

$$h(x)=\frac{\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}}{1-(1-\gamma)e^{-\left(\frac{x}{\alpha}\right)^{\beta}}} \tag{8}$$

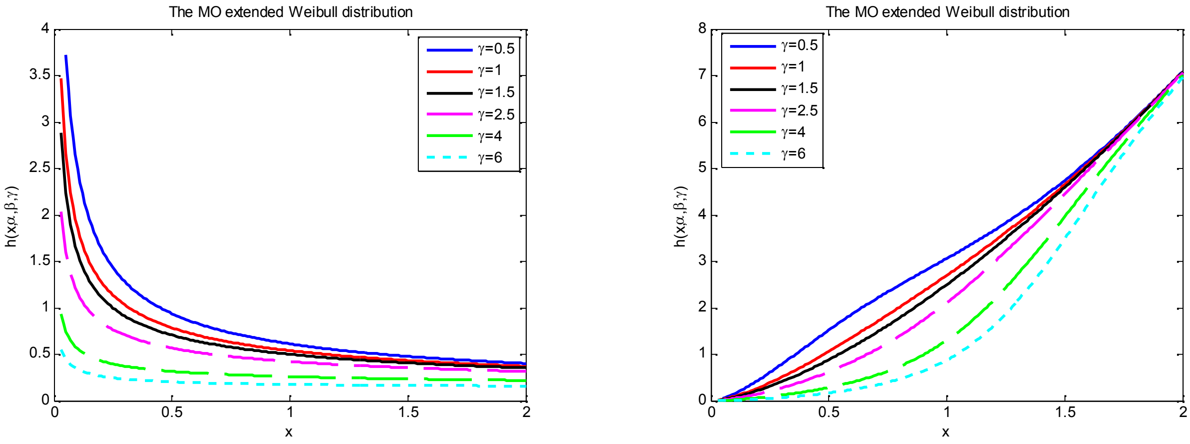
The pdf, given in Eq. 7, is unimodal if $\beta\leq 1$, $\gamma>1$ or $\beta>1$, $\gamma>1$ and it is also decreasing if $\beta\leq 1$, $\gamma\leq 1$ (Zhang and Xie, 2007).

In Fig. 1, some possible shapes of the pdf of MOEW distribution are illustrated. Additionally, Fig. 2 shows plots of the hazard function of MOEW distribution, which can have increasing, decreasing, increasing–decreasing–increasing and decreasing – increasing– decreasing forms.



(a) **(b)**

Fig. 1: Plots of the pdf of MOEW distribution for α=1, **(a)** γ=0.5, **(b)** γ=1.5 and different values of β.



(a) **(b)**

Fig. 2: Graphs of the hazard function of MOEW distribution for α=1, (a) β=0.5, (b) β=2.5 and different values of γ.

We can easily invert the cdf, in Eq. (6), to obtain the MOEW quantile function,

$$x = Q(u) = \alpha\left\{\ln\left(\frac{1-(1-\gamma)u}{1-u}\right)\right\}^{1/\beta} \quad (9)$$

where $0<u<1$. Generation of the MOEW random variable follows easily from Eq. (9), i.e. if U form(0,1), then X [redacted]) has a MOEW distribution. In particular, the median of MOEW distribution is given as follows,

$$\text{median} = \alpha\{\ln(1+\gamma)\}^{1/\beta} \quad (10)$$

Considering the series expansion,

$$(1-z)^{-k} = \sum_{j=0}^{\infty}\frac{\Gamma(k+j)z^j}{\Gamma(k)j!} \quad (11)$$

where $0<u<1, k>0$, and $\Gamma(.)$ is gamma function. For $\gamma\in(0,1)$, the *r*th moment of MOEW distribution can be written as

$$E(X^r) = \alpha^r\Gamma\left(\frac{r}{\beta}+1\right)\sum_{j=0}^{\infty}\frac{w_j}{(j+1)^{r/\gamma}} \quad (12)$$

where $w_j = \gamma(1-\gamma)^j$. If $\gamma>0$, the *r*th moment can be given as

$$E(X^r) = \alpha^r\Gamma\left(\frac{r}{\beta}+1\right)\sum_{j=0}^{\infty}\frac{v_j}{(j+1)^{r/\gamma}} \quad (13)$$

where $v_j = [(j+1)\gamma]^{-1}\sum_{j=0}^{\infty}(-1)^j(k+1)\binom{j}{k}(1-1/\gamma)^k$.

More statistical properties of the MOEW distribution are discussed in Cordeiro and Lemonte (2013).

3. Estimation methods

In this section, we present the maximum likelihood (ML), least squares (LS) and weighted least square (WLS) methods used to obtain the estimates for the three parameters of MOEW distribution.

3.1. Maximum likelihood method

The ML method is one of the most effective and commonly-used methods for parameter estimation (Usta, 2013). The ML estimates of parameters can be obtained by maximizing the log-likelihood function. In this case, for a sample $X=(X_1,X_2,\ldots,X_n)$ of size n from the MOEW distribution, the log-likelihood function can be written as:

$$\ln L(\quad) = \left(\frac{\gamma\beta}{\alpha}\right) + (\ -\ )\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta} - \sum_{i=1}^{n}\quad -(\ -\ )^{-\left(\frac{x_i}{\alpha}\right)^{\beta}} \quad (14)$$

By differentiating Eq. (14) partially with respect to α, β and γ, the normal equations are obtained as follows:

$$\frac{\partial \log L}{\partial \alpha} = -\frac{n}{\alpha} - -\frac{}{\alpha}\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta} - 2(1-\ )\frac{}{\alpha}\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta} e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}}{1-(1-\ )^{\left(\frac{-x_i}{\alpha}\right)^{\beta}}} \quad (15)$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\ln\left(\frac{x_i}{\alpha}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right) - 2(1-\ )\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right)e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}}{1-(1-\ )^{\left(\frac{-x_i}{\alpha}\right)^{\beta}}} \quad (16)$$

$$\frac{\partial \log L}{\partial \gamma} = \frac{n}{\gamma} - 2\sum_{i=1}^{n} \frac{e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}}{1-(1-\gamma)e^{-\left(\frac{x_i}{\alpha}\right)^{\beta}}} \quad (17)$$

Equating these nonlinear equations to zero, the ML estimates of α, β and γ can be obtained. Since the system of Eqs. (15)–(17) doesn't have an analytic solution, these estimates can be achieved by using numerical methods such as the Newton-Raphson method.

3.2. Least squares and weighted least square methods

The LS and WLS methods were proposed by Swain et al. (1988) to estimate the parameters of Beta distributions. In this sense, the LS and WLS estimates are obtained, respectively, by minimizing the functions:

$$\sum_{i=1}^{n}[F(x_{(i)}) - E[F(x_{(i)})]]^2 \quad (18)$$

and

$$\sum_{i=1}^{n}\frac{1}{var[F(x_{(i)})]}[F(x_{(i)}) - E[F(x_{(i)})]]^2 \quad (19)$$

with respect to the unknown parameters. Assume that $X = (X_1, X_2, ..., X_n)$ is a random sample of size n from a cdf, $F(x)$, and $(x_{(1)} < x_{(2)} < ... < x_{(n)})$ denote ordered random variables. Also, it is well-known that

$$E[F(x_{(i)})] = \frac{i}{n+1}, \quad Var[F(x_{(i)})] = \frac{i(n-i+1)}{(n+1)^2(n+2)} \quad (20)$$

In the case of the MOEW distribution, considering Eqs. (18)–(20) and the cdf, given in Eq. (6), the LS estimates of unknown three parameters of the MOEW distribution are derived by minimizing the functions:

$$\sum_{i=1}^{n}\left(\frac{1-e^{-\left(\frac{x_{(i)}}{\alpha}\right)^{\beta}}}{1-(1-\gamma)e^{-\left(\frac{x_{(i)}}{\alpha}\right)^{\beta}}} - \frac{i}{n+1}\right)^2 \quad (21)$$

with respect to α, β and γ.

The WLS estimates of parameters α, β and γ are obtained by minimizing the functions:

$$\sum_{i=1}^{n}\frac{i(n-i+1)}{(n+1)^2(n+2)}\left(\frac{1-e^{-\left(\frac{x_{(i)}}{\alpha}\right)^{\beta}}}{1-(1-\gamma)e^{-\left(\frac{x_{(i)}}{\alpha}\right)^{\beta}}} - \frac{i}{n+1}\right)^2 \quad (22)$$

with respect to α, β and γ. It is emphasized that some non-linear optimization techniques should be used to minimize Eqs. (21) and (22).

4. Numerical experiments

In this section, a simulation study is conducted to compare the performance of the different estimators that are explained in the previous section. The design of simulation study is outlined in the following. After a description of the simulation steps, the obtained results are presented.

The steps of simulation study are as follows:

i) Given the parametersα, β and γ, generate a sample $(x_1, x_2, ..., x_n)$ of size n from the MOEW distribution using the transformation $x = \left\{ \left(\frac{1-(1- \quad)}{1-U}\right)\right\}^{1/}$, U has Uniform distribution over the interval (0,1).

ii) Compute the parameter estimates with the different methods considered in Section 3. The ML estimates of the parameters α, β and γ, can be obtained by solving the nonlinear Eqs. (15)-(17). The LS and WL estimates of the parameters α, β and γ, obtained by minimizing the Eqs. (21) and (22).

iii) Given size n=20, 50 and 100, repeat first and second steps N times where N is taken as 10000/n.

iv) Compare the parameter estimates $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$, with the true parameters α, β and γ, by computing the bias and MSE defined as :

$$Bias(\hat{\theta}) = \theta - \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_i \quad (25)$$

$$MSE(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_i - \theta\right)^2 \quad (26)$$

where $\hat{\theta}_i$, i=1,...,N, are N estimates of θ and N is the number of simulation replications.

During the simulation study, the considered methods have been compared for β=0.5, 1.5, 2.5 and γ=0.5, β=0.5, 1.5, 2.5 and γ=1.5, β=0.5, 1.5, 2.5 and γ=3. When the simulation with different α value was repeated, it is observed that the results were invariant so the parameter α was taken as 1 in all considered cases.

In this section, the results of the simulation study are presented to ascertain the performance of the considered estimators of the parameters α, β and γ. The obtained results are reported as the bias and MSE of the considered methods in Tables 1-3 for α=1, γ=0.5 and β=0.5,1.5,2.5, α=1, γ=1.5 and β=0.5,1.5,2.5, α=1, γ=3 and β=0.5,1.5,2.5, respectively .

Table 1: Simulation results for α=1, γ=0.5 and different values of β

β=0.5		n=20		n=50		n=100	
		Bias	MSE	Bias	MSE	Bias	MSE
ML	α	0.0142	0.4850	0.0184	0.4135	0.0763	0.3207
	β	0.0239	0.0179	0.0035	0.0076	-0.0011	0.0048
	γ	0.2249	0.3470	0.2199	0.2930	0.1634	0.2292
LS	α	0.2933	0.6901	0.2213	0.5903	0.2034	0.5305
	β	-0.0225	0.0159	-0.0098	0.0084	-0.0050	0.0046
	γ	0.1822	0.2813	0.1333	0.2007	0.0858	0.1200
WLS	α	0.4607	0.4323	0.3787	0.3617	0.2495	0.2243
	β	0.0111	0.0165	0.0127	0.0075	0.0089	0.0035
	γ	-0.0361	0.0850	-0.0418	0.0571	-0.0334	0.0339
β=1.5		n=20		n=50		n=100	
		Bias	MSE	Bias	MSE	Bias	MSE
ML	α	0.0970	0.2058	0.0465	0.1305	0.0391	0.0914
	β	0.1687	0.2179	0.0606	0.0906	0.0304	0.0530
	γ	0.1358	0.3401	0.1539	0.2957	0.1107	0.2244
LS	α	0.1530	0.3631	0.1106	0.2490	0.0777	0.1573
	β	-0.0225	0.1656	-0.0225	0.1024	-0.0149	0.0650
	γ	0.2970	0.5989	0.2481	0.4936	0.1844	0.3663
WLS	α	-0.0400	0.1616	-0.0213	0.0943	0.0197	0.0724
	β	-0.1151	0.1127	-0.0647	0.0585	-0.0220	0.0378
	γ	0.3765	0.4371	0.2274	0.2391	0.1114	0.1322
β=2.5		n=20		n=50		n=100	
		Bias	MSE	Bias	MSE	Bias	MSE
ML	α	0.0037	0.0586	0.0052	0.0426	0.0041	0.0294
	β	0.1840	0.4318	0.0932	0.2483	0.0379	0.1449
	γ	0.1936	0.3774	0.1681	0.3039	0.1320	0.2288
LS	α	-0.0182	0.0990	-0.0084	0.0662	0.0051	0.0484
	β	-0.1338	0.4035	-0.1041	0.2432	-0.0575	0.1673
	γ	0.4109	0.6375	0.3173	0.4966	0.2255	0.3797
WLS	α	0.0261	0.1079	-0.0230	0.0431	-0.0041	0.0297
	β	-0.1764	0.2000	-0.1216	0.1142	-0.0659	0.0798
	γ	0.3051	0.3566	0.2387	0.2264	0.1438	0.1486

According to the bias, for α, it can be seen from Table 1 that the ML method shows the best performance for all sample sizes when β=0.5, 2.5. The WLS method outperforms other methods for β=1.5 at all sample sizes. Regarding β, the bias of ML method are lower than other methods for β=0.5, 2.5 when the sample sizes are 50 and 100. However, for β=1.5, the LS is observed as a best method for all sample sizes. On the other hand, for γ, the ML method has minimum bias when the parameter values of β are 1.5 and 2.5 for all sample sizes, but when β=0.5, the WLS gives the best results. Regarding the MSE, for α, while the WLS method exhibits the best performance relative to β=0.5, 1.5 for all sample sizes, the MLE performs well for β=2.5. Additionally, for β and γ, the WLS becomes a best method in terms of MSEs for all values of β and all sample size except for β=0.5, n=20 and β=1.5, n=20.

Table 2: Simulation results for α=1, γ=1.5 and different values of β

β=0.5		n=20		n=50		n=100	
		Bias	MSE	Bias	MSE	Bias	MSE
ML	α	0.1970	0.6002	0.3059	0.5479	0.2726	0.5191
	β	0.0457	0.0187	0.0332	0.0088	0.0202	0.0056
	γ	0.1178	0.7226	-0.0083	0.5871	0.0081	0.5356
LS	α	0.0702	0.4592	0.0728	0.4001	0.0976	0.3357
	β	-0.0180	0.0139	-0.0047	0.0068	-0.0018	0.0049
	γ	0.2919	0.7353	0.2490	0.6120	0.1568	0.4712
WLS	α	-0.1778	0.2646	-0.1854	0.2115	-0.1194	0.1962
	β	-0.0351	0.0118	-0.0258	0.0055	-0.0216	0.0034
	γ	0.5622	0.8194	0.5111	0.6575	0.3762	0.4980
β=1.5		n=20		n=50		n=100	
		Bias	MSE	Bias	MSE	Bias	MSE
ML	α	0.1469	0.1273	0.1105	0.0892	0.0831	0.0586
	β	0.3062	0.3187	0.1489	0.1175	0.0997	0.0644
	γ	-0.1927	0.7434	-0.0945	0.6448	-0.0431	0.5891
LS	α	-0.0619	0.1074	-0.0471	0.0776	-0.0511	0.0553

	β	-0.0535	0.1832	-0.0527	0.0892	-0.0546	0.0554
	γ	0.5484	1.0710	0.4403	0.9603	0.4534	0.9110
WLS	α	-0.1500	0.0528	-0.1139	0.0289	-0.0776	0.0191
	β	-0.1387	0.1179	-0.1000	0.0536	-0.0691	0.0270
	γ	0.7892	0.9069	0.5568	0.5177	0.3964	0.3302
β=2.5		**n=20**		**n=50**		**n=100**	
		Bias	**MSE**	**Bias**	**MSE**	**Bias**	**MSE**
ML	α	0.0718	0.0423	0.0594	0.0301	0.0371	0.0216
	β	0.3342	0.5010	0.2467	0.3255	0.1418	0.1869
	γ	-0.1008	0.7213	-0.0684	0.6766	0.0178	0.6126
LS	α	0.0588	0.0695	0.0050	0.0340	-0.0016	0.0247
	β	-0.0140	0.3440	-0.0428	0.2511	-0.0281	0.1698
	γ	0.1013	0.9561	0.2703	0.8775	0.2766	0.8087
WLS	α	-0.0883	0.0187	-0.0582	0.0098	-0.0443	0.0067
	β	-0.2643	0.2688	-0.1677	0.1176	-0.1203	0.0685
	γ	0.7274	0.8058	0.4813	0.4440	0.3677	0.3095

When the parameters α, γ and β are considered as α=2, γ=1.5 and β=0.5,1.5,2.5, the results with regard to bias in Table 2 then point out that for α and β, the LS method provides smaller biases than the others for all sample sizes. For γ, the ML has minimum bias at all sample cases. Considering the MSEs, the WLS method gives the best performance for α and β at all sample sizes. Besides, for γ, the ML outperforms other methods for n=20 and all cases of β. When n=50, 100, the WLS method is the best in terms of the MSE.

Table 3: Simulation results for α=1, γ=3 and different values of β

β=0.5		**n=20**		**n=50**		**n=100**	
		Bias	**MSE**	**Bias**	**MSE**	**Bias**	**MSE**
ML	α	0.2723	0.4874	0.3373	0.5240	0.3644	0.5339
	β	0.0474	0.0166	0.0379	0.0091	0.0324	0.0057
	γ	-0.2968	1.2147	-0.2580	1.1925	-0.3445	1.1313
LS	α	-0.0731	0.2561	-0.0356	0.1653	-0.0361	0.1107
	β	-0.0223	0.0135	-0.0133	0.0054	-0.0104	0.0027
	γ	0.2503	0.6980	0.2312	0.6390	0.1961	0.5451
WLS	α	-0.1393	0.1824	-0.1020	0.0832	-0.1120	0.0563
	β	-0.0278	0.0123	-0.0160	0.0049	-0.0157	0.0025
	γ	0.4836	0.5132	0.3928	0.3255	0.3411	0.2496
β=1.5		**n=20**		**n=50**		**n=100**	
		Bias	**MSE**	**Bias**	**MSE**	**Bias**	**MSE**
ML	α	0.0847	0.0652	0.0840	0.0386	0.0756	0.0316
	β	0.1878	0.1998	0.1289	0.0796	0.0978	0.0527
	γ	-0.2936	1.3378	-0.3572	1.1278	-0.2638	1.0703
LSE	α	-0.0862	0.0524	-0.0610	0.0330	-0.0503	0.0249
	β	-0.1387	0.1295	-0.0657	0.0630	-0.0602	0.0417
	γ	0.4785	1.1969	0.5092	1.1151	0.4782	1.0396
WLS	α	-0.0976	0.0357	-0.0637	0.0162	-0.0491	0.0092
	β	-0.1068	0.1182	-0.0576	0.0505	-0.0491	0.0265
	γ	0.6478	0.7150	0.5079	0.4119	0.3965	0.2938
β=2.5		**n=20**		**n=50**		**n=100**	
		Bias	**MSE**	**Bias**	**MSE**	**Bias**	**MSE**
ML	α	0.0501	0.0192	0.0424	0.0141	0.0509	0.0123
	β	0.3244	0.4472	0.1884	0.2149	0.1805	0.1578
	γ	-0.2740	1.1360	-0.2647	1.0956	-0.3566	1.0154
LS	α	-0.0126	0.0235	0.0001	0.0154	-0.0150	0.0093
	β	-0.0780	0.3131	-0.0486	0.1576	-0.0569	0.0973
	γ	0.0954	1.2567	0.1070	1.1728	0.2857	0.9766
WLS	α	-0.0486	0.0122	-0.0346	0.0055	-0.0257	0.0032
	β	-0.1097	0.3126	-0.1027	0.1310	-0.0732	0.0683
	γ	0.5519	0.5490	0.4145	0.3072	0.3279	0.2333

As can be seen in Table 3, for all sample sizes, the LS method provides the smallest bias for estimating α, β and γ when the parameter β is considered as 0.5 and 2.5. For β=1.5, the WLS and ML methods show good performance for β and γ, respectively. The computed MSEs indicated that the WLS method can be regarded as the best method to estimate the parameters α, β and γ for all sample sizes.

5. Conclusion

In this study, we compared the performance of maximum likelihood (ML), least squares (LS) and weighted least squares (WLS) methods for estimating the parameters of Marshal-Olkin extended Weibull (MOEW) distribution. These comparisons were conducted in terms of bias and mean square error (MSE) by an extensive numerical simulation.

On the basis of the simulation results, it can be concluded that in terms of e bias, the LS method usually outperforms the other methods for α and β at most of sample sizes, while the ML generally shows good performance for γ. Moreover, the WLS method provides the comparable performance for estimating the parameters α, β and γ.

Comparing the MSE, the WLSE method mostly outperforms the other estimators for α, β and γ at all sample cases. In pursuit of the WLSE method, the ML usually comes next.

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