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Eigenvalue assignment in state feedback control for uncertain systems

M. Rahimi [1,*], A. Naserpour [1], S. M. Karbassi [2]

[1]Department of Mathematics, College of Engineering, Fereydan Branch, Islamic Azad University, Esfahan, Iran
[2]Department of Mathematics, University of Yazd, Yazd, Iran

Abstract: This paper presents a method for assignment the eigenvalues by state feedback for uncertain linear systems. This eigenvalue assignment is based on Karbassi-Bell method. The main advantage of this method is that we can parameterizations of the state feedback control for linear multivariable systems and we can use of this form to find optimal solution. For optimization of this problem we propose a genetic algorithm.

Key words: State feedback; Uncertain systems; Eigenvalue assignment

1. Introduction

Many systems in the real world cannot be modeled with great accuracy. There are often uncertainties, with can arise from unmodelled dynamics manufacturing tolerances. In complete knowledge of physical system. Therefore a more realistic approach, in general is to model the systems concerned with a few uncertain parameters whose values are not known exactly. But for which lower and upper bounds are known. Several control systems design methods such the generalized Kharitunov approach (Ackermann and Barmish, 1993) have been developed to.address such problems

The design of feedback controllers for structured uncertain systems to assign the eigenvalue of closed -loop system matrix in a specified region is on important problem.

In this paper we use of Karbassi-Bell method for the parameterizations of state feedback controllers. A very interesting outcome of this study is that the nonlinear system of equations for eigenvalue assignment for a given pair of system matrices and a given set of eigenvalues is uniquely determined by the structural properties of the systems that is the Kroneckerinvariants as defined in Karbassi (1993) For optimization of this problem we propose a genetic algorithm (Goldberg, 1989).

2. The Basic Results

Consider a linear MIMO structured uncertain systems described by the state equations:

$\dot{x}$=Ax+Bu y=Cx

Where A,B,C have dimensions n×n ,n×mand l×n respectively and x∈R^n , u∈R , y∈R . It is also assumed that the pair (A, B) is controllable. The aim of eigenvalue assignment is to design a state feedback controller producing closed–loop systems with a satisfactory response by shifting controllable eigenvalue from undesirable to desirable locations.

This means that K is chosen such that the eigenvalue of closed–loop systems r=A+BK lie in the self-conjugate eigenvalue spectrum $A=\{\lambda_1,\dots,\lambda_1\}$.

More formally an uncertain system description canbegiven in the following form:

$\dot{x}$= A (q)x+B(q) u y= C(q)x

where x∈R^n ,u∈R , y∈Rare state input and output vectors, respectively and the state space matrices A,B,C are function of uncertainty vector q; $q=\{q_1,\dots,q_n\}$

Whose elements are known to be interval uncertain parameters?

I.e. $q_i^- \le q_i \le q_i^+$ for i=1, 2, ..., r

The first step we find the general form of compensators that assign the nominal plant eigenvalues F (K) by Karhassi-Bell method. Then calculate the closed–loop characteristic polynomial as a function of the parametric uncertainties (q) free controller variable (k) and the frequency variable(s), p_c(s, q, k) or the general form of the closed–loop state matrix A_c(q, k). The final step is find k=k* such that p(s, q, k*) be an optimization of this problem.

3. Main Results

Consider a controllable linear time–invariants system defined by the state equation $\dot{x}$ =Ax(t)+Bu(t).consider the state transformation x(t)=T $\tilde{x}$ (t) or $\tilde{x}$ (t)=T^{-1}x(t)

Where T can be obtained by elementary similarity operations as described in [5, 6].

In this way, $\tilde{A}$ =T^{-1}AT and $\tilde{B}$ =T^{-1}B is in compact canonical form known as vector companion form:

$$\tilde{A}=\begin{bmatrix} G_o \\ I_{n-m}, O_{n-m,m} \end{bmatrix}, \tilde{B}=\begin{bmatrix} B_o \\ O_{n-m,m} \end{bmatrix}$$

* Corresponding Author.

Where G_o is an m×n matrix and B_o is an m×m upper triangular matrix. Note that if the Kronecker invariants of the pair (B,A) are regular , then $\tilde{A}$ and $\tilde{B}$ are always in the above form, we may also conclude that if the vector companion form $\tilde{A}$ obtained from similarity operations has the above structure then the Kronecker invariants associated with the pair (B,A) are regular.

The state feedback matrix which assigns all he eigenvalues to zero for the transformed pair ($\tilde{A}$, $\tilde{B}$) is then chosen as u=$B_0^{-1}G_0\tilde{x} = \tilde{F}\tilde{x}$

Which results in the primary state feedback matrix for the pair (B, A) defined as $F_p = \tilde{F}T^{-1}$

The transformed closed–loop matrix $\tilde{\Gamma}_0$=A+BF assumes a compact Jordan form with zero eigenvalue.

$$\tilde{\Gamma}_o = \begin{bmatrix} O_{\min} \\ \mathrm{I}_{n-m}, O_{n-m,m} \end{bmatrix}$$

Now let $\tilde{A}_\lambda$ be any matrix in vector companion form, i.e.

$$\tilde{A}_\lambda = \begin{bmatrix} G_\lambda \\ \mathrm{I}_{n-m}, O_{n-m,m} \end{bmatrix}$$

With the eigenvalue spectrum $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ a set of self–conjugate eigenvalues. Then $\tilde{\Gamma} = \tilde{A} + \tilde{B}$.

Furthermore if $\tilde{K}$ is the controller matrix which assigns the set self-conjugate eigenvalues $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ to transformed pair ($\tilde{A}$,$\tilde{B}$) then $K = \tilde{K}T^{-1} = B_0^{-1}(-G_o + G_\lambda)T^{-1}$

Is the controller matrix which assigns the same set of eigenvalue to the pair (B, A). For more details the interested reader is referred to (Karbassi, 1994)

Finally the parametric feedback matrix is defined as $K = K_P + K_\alpha$

Where $K_\alpha = B_o^{-1}GT^{-1}$ and G is an m×n matrix containing free parameters only. The simplest way to locate the parameters is the method for state transition graph (Karbassi, 2001).

Applied to the compact Jordan form. Thus consider the transition of unit vectors {e$_1$, e2, …, em} as inputs by Γ as the closed–loop matrix and graph representing this transition. The free parameters are specified as the edges joining the vertices such that no two paths coincide or interest each other.

3.1. Example

Consider the following system where the A and B matrices are given as

$$As = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1+q_1 & 2+q_2 \\ 2 & 1 & -2 \end{bmatrix} ,$$

$$Bs = \begin{bmatrix} 0 & 1 \\ -1+q_3 & 0 \\ 1 & 1 \end{bmatrix}$$

Where the qi are interval uncertain parameters bounded as follows Stem.

And have nominal values $q_{1n} = q_{2n} = q_{3n} = o$ eigenvalues of the nominal system therefore are {-3.284, o.642+o.897i} now suppose the closed–loop system values under nominal working conditions are required to be at {-7, -2 ±i}.

By Karbassi–Bell method for nominal system we have $K_p = \begin{bmatrix} -12 & 22 & 16 \\ -2 & 0 & -1 \end{bmatrix}$ and

$K_\alpha = \begin{bmatrix} -k_1+k_2 & -k_1-k_2 & -k_2 \\ -k_1+k_2 & -k_1-k_2 & -k_2 \end{bmatrix}$ then we have

$$K = K_\alpha + K_p = \begin{bmatrix} -12-k_1+k_2 & 22-k_1-k_2 & 16-k_2 \\ -2-k_1+k_2 & -k_1-k_2 & -1-k_2 \end{bmatrix}$$

The general form of the closed-loop systems characteristic:

$$P(s,q,k) = s^3 + (k_1q_3 + q_3k_2 - 22q_3 - q_1 + 11)s^2 + (-15k_1q_3 - k_1q_1 + 10q_3k_2 - k_2q_1 + 2k_2q_2 + 2k_1q_2 + 5k_1 - 116q_3 + 12q_1 - 23q_2 + 33)s + (-85k_1q_3 + 75k_1q_1 + 30q_3k_2 + 5k_2q_1 + 15k_2q_2 - 35k_1q_2 - 130q_3 + 5k_2 + 25q_1 - 35q_3 + 35)$$

Using the genetic algorithm the most robust compensator is obtained for $k_1 = -0.543, k_2 = -1.843$in with case the state-feedback matrix is givens as

$$K = K_\alpha + K_p = \begin{bmatrix} -13.3 & 24.386 & 17.843 \\ -3.3 & 2.386 & 0.843 \end{bmatrix}$$

4. Conclusions

In this paper we use the genetic algorithm for assignment the eigenvalues by state feedback in uncertain linear systems. This eigenvalue assignment is based on Karbassi-Bell method.The main advantage of this method is that we can parameterizations of the state feedback control for linear multivariable systems by graph theory and we can use of this form to find optimal solution. For optimization of this problem we propose a genetic algorithm.

The algorithm presented is simpler and more concise than the existing methodologies of Ackermann (1993), Munro and Soylemez (1997).

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